Spectral methods toward correspondence-free geometric deep learning

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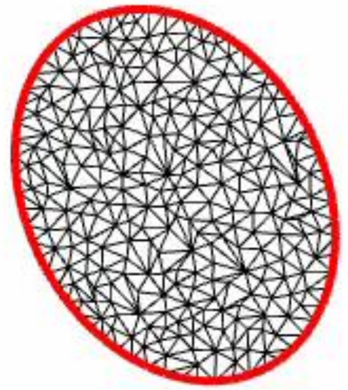

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CAN ONE HEAR THE SHAPE OF A DRUM?

MARK KAC, The Rockefeller University, New York

To George Eugene Uhlenbeck on the occasion of his sixty-fifth birthday

> "La Physique ne nous donne pas seulement l'occasion de résoudre des problèmes . . . , elle nous fait presentir la solution." H. POINCARÉ.

Wave equation

$$\frac{\partial^2 u(x,y;t)}{\partial t^2} = \Delta u(x,y;t)$$

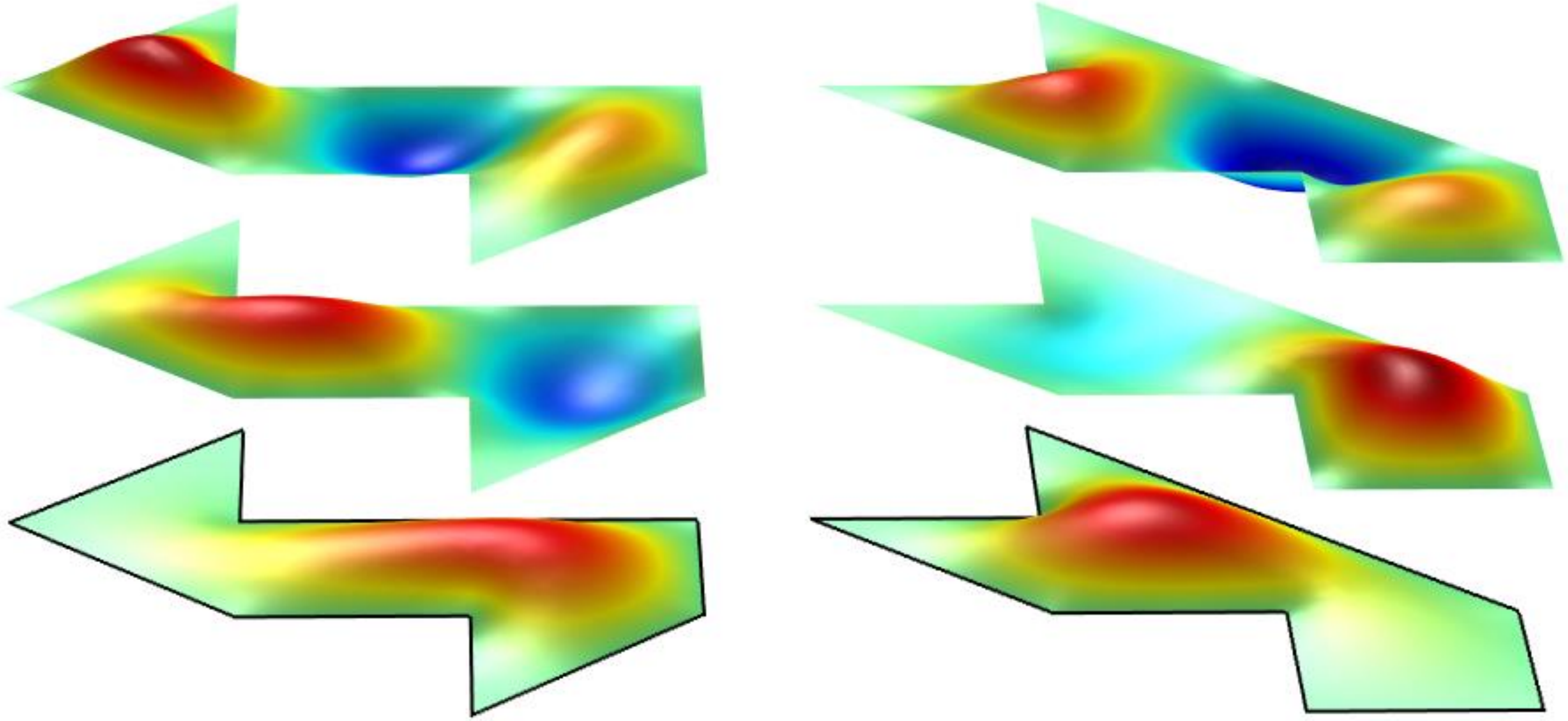
$$\Delta \phi_i = \lambda_i \phi_i$$

$$u(x,y;t) = \sum_j d_j \phi_j(x,y) \left(\cos\left(\sqrt{\lambda_j} t\right) + i \sin\left(\sqrt{\lambda_j} t\right) \right)$$

Isometry invariance

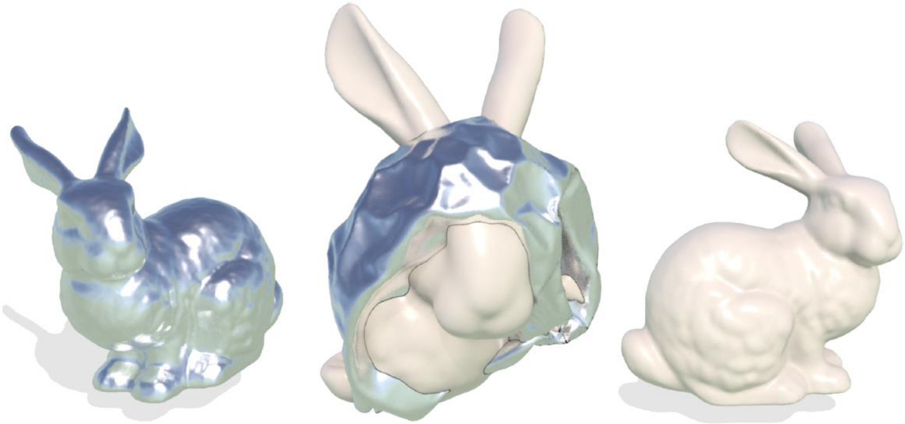
Isometric shapes have the same Laplacian eigenvalues

Isospectral $\neq$ Isometric



Existing approaches

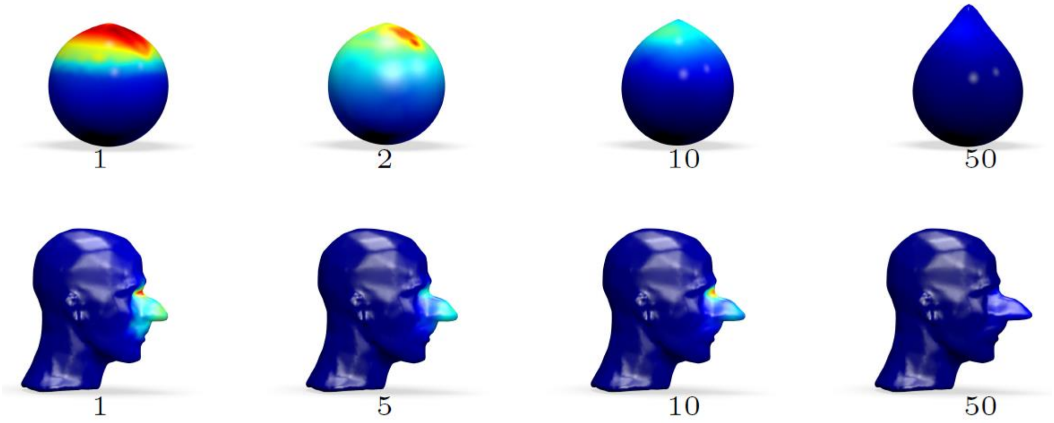
Shape-from-metric



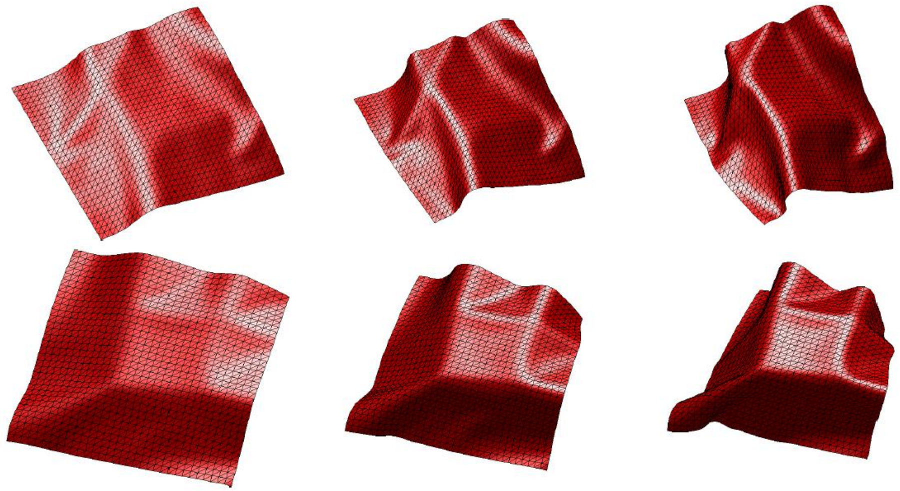
Chern et al 2018

Borrelli et al 2012

Shape-from-operator



Boscaini et al 2014



Corman et al 2017

An empirical approach

Discrete setting

Isospectralization

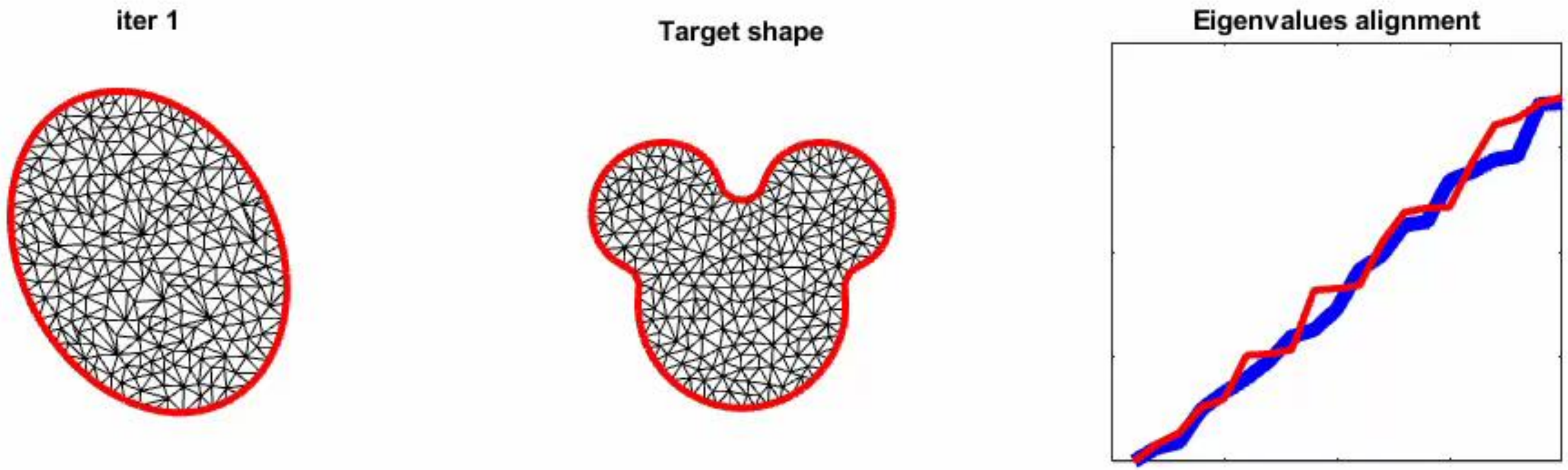
$$\min_{\mathbf{X}\in\mathbb{R}^{n\times d}} \|\boldsymbol{\lambda}(\boldsymbol{\Delta}_X(\mathbf{X})) - \boldsymbol{\mu}\|_\omega + \rho_X(\mathbf{X})$$

- Data term: weighted norm (frequency balancing)

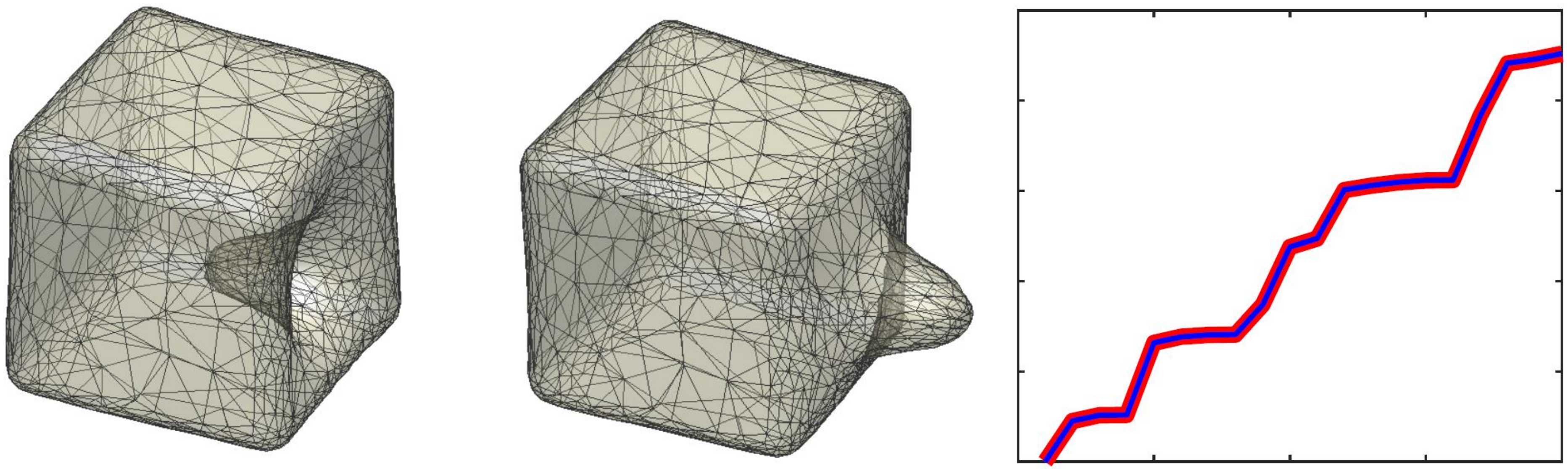
$$\|\boldsymbol{\lambda} - \boldsymbol{\mu}\|_\omega^2 = \sum_{i=1}^{k} \frac{1}{\mu_i^2}(\lambda_i - \mu_i)^2$$

- Regularizers to promote smoothness / maximize volume
- Input: ≤ 30 eigenvalues
- Optimization: Nonlinear conjugate gradient with automatic differentiation

Example: Mickey-from-spectrum

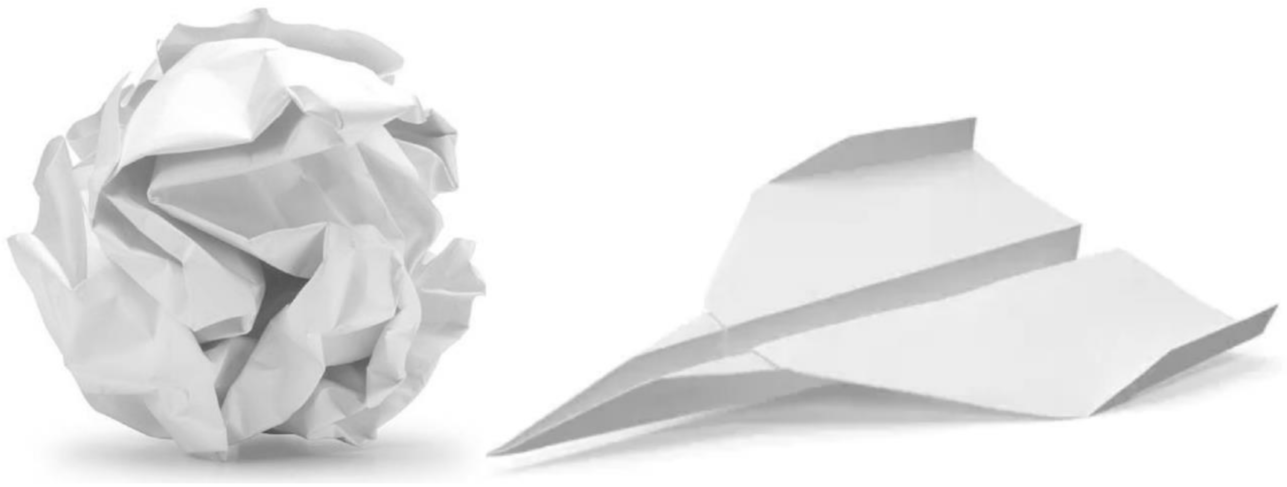


Geometric priors



Example: **Volume** regularizer to avoid isometric ambiguities

Geometric priors are not enough



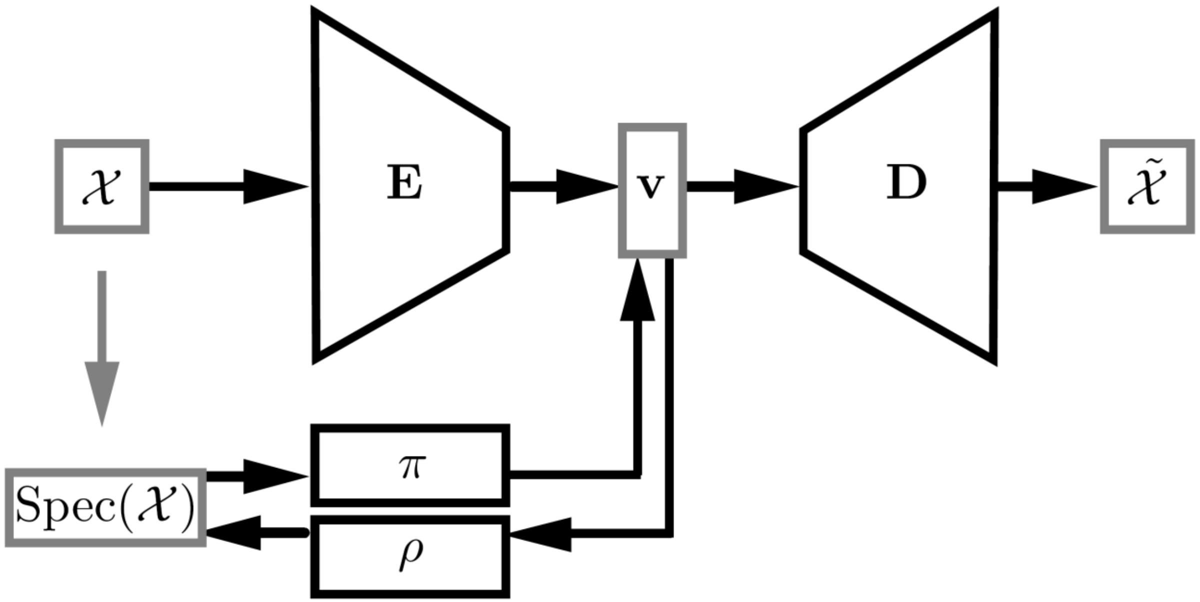
Do we really have to design regularizers?

Learn from data what is hard to model axiomatically

Data-driven formulation

Latent space connections

AE-based learning model:



Latent space connections

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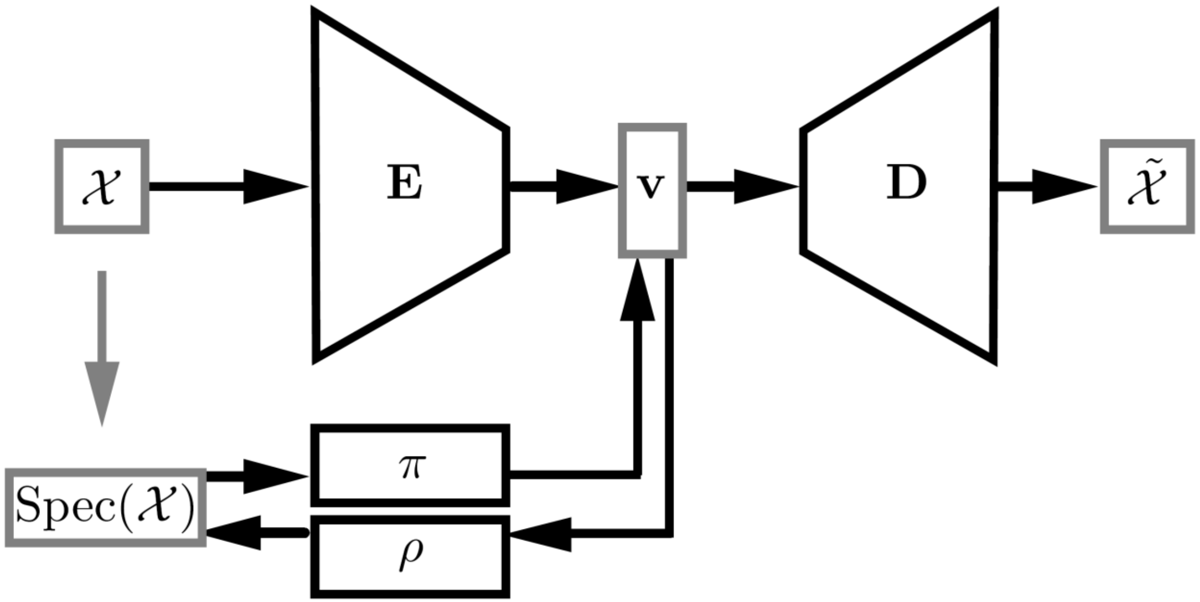

$$\ell = \ell_{\mathcal{X}} + \alpha \ell_{\lambda}\,, \quad \text{with}$$

$$\ell_{\mathcal{X}} = \frac{1}{n}\|D(E(\mathbf{X})) - \mathbf{X}\|_F^2$$

$$\ell_{\lambda} = \frac{1}{k}(\|\pi(\boldsymbol{\lambda}) - E(\mathbf{X})\|_2^2 + \|\rho(E(\mathbf{X})) - \boldsymbol{\lambda}\|_2^2)$$

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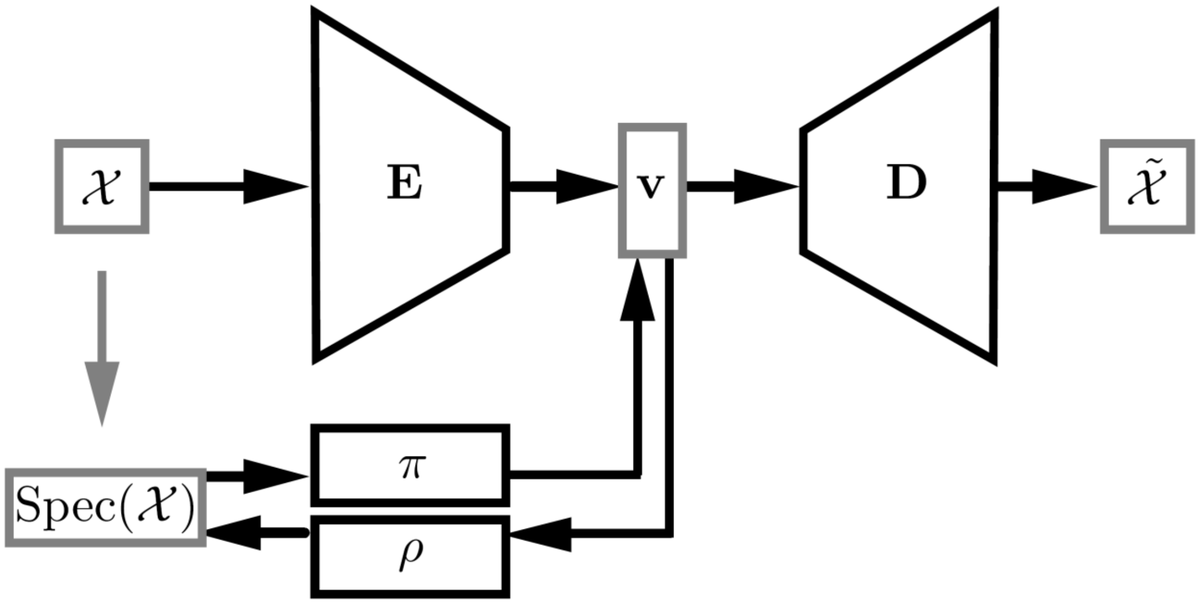

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The spectral loss enforces:

$$\rho \approx \pi^{-1}$$

Latent space connections

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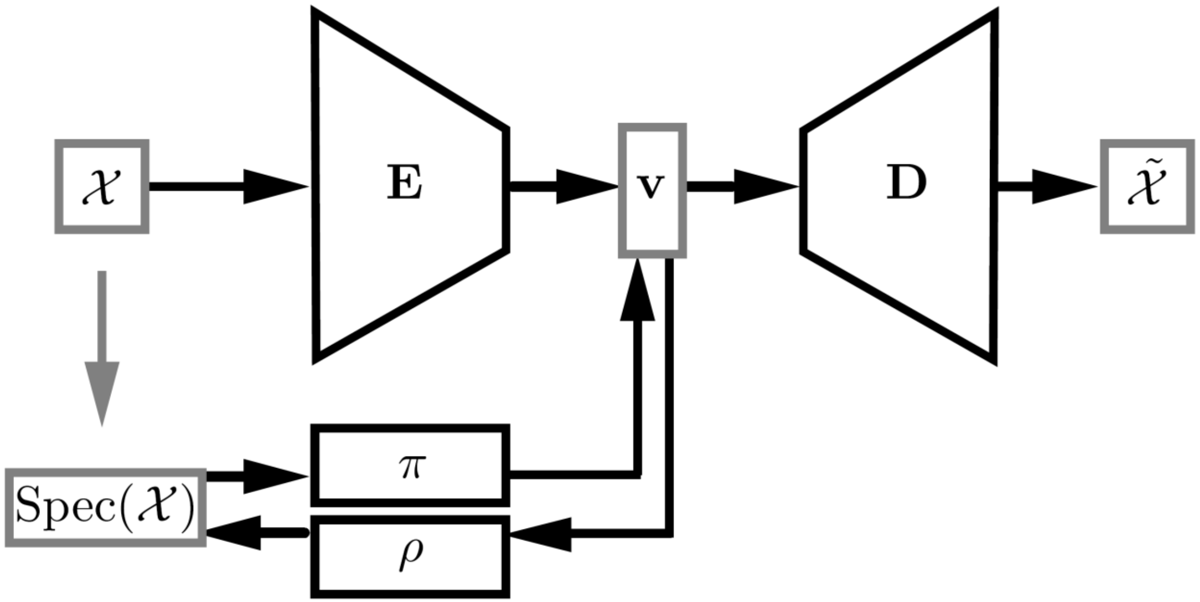

Remarks:

- **No back-propagation** through the eigen-decomposition

Latent space connections

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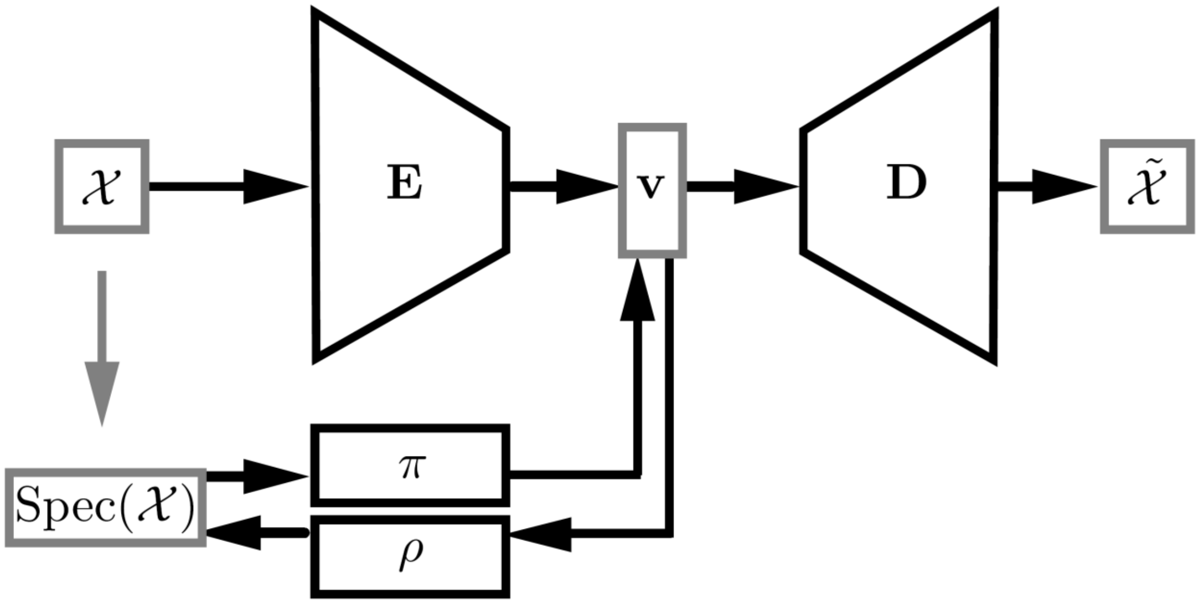

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Latent space connections

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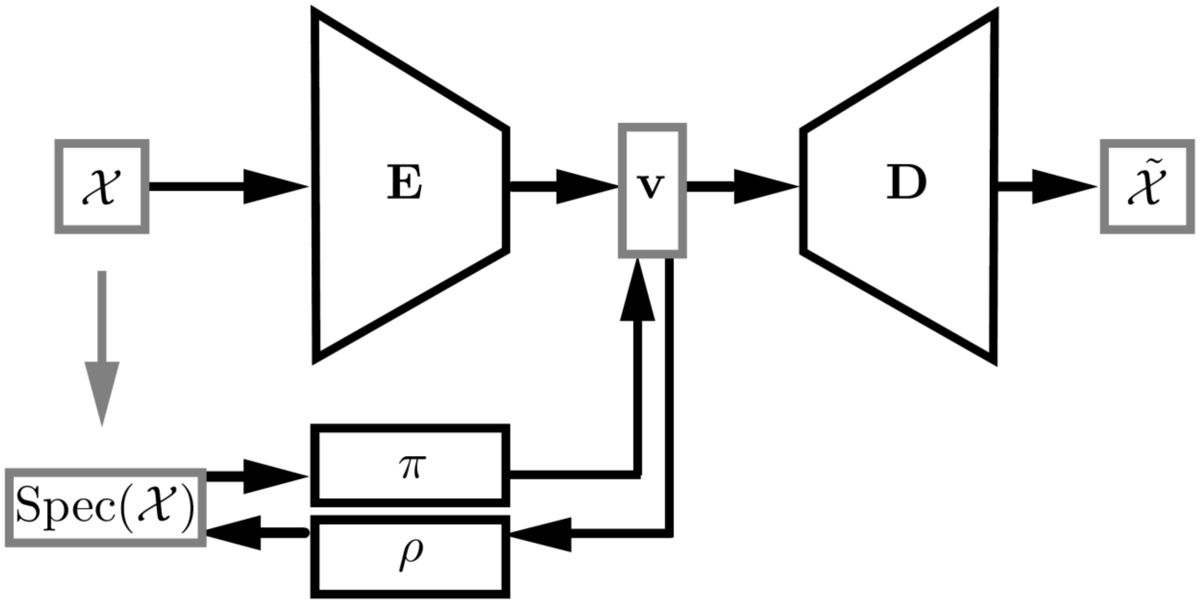
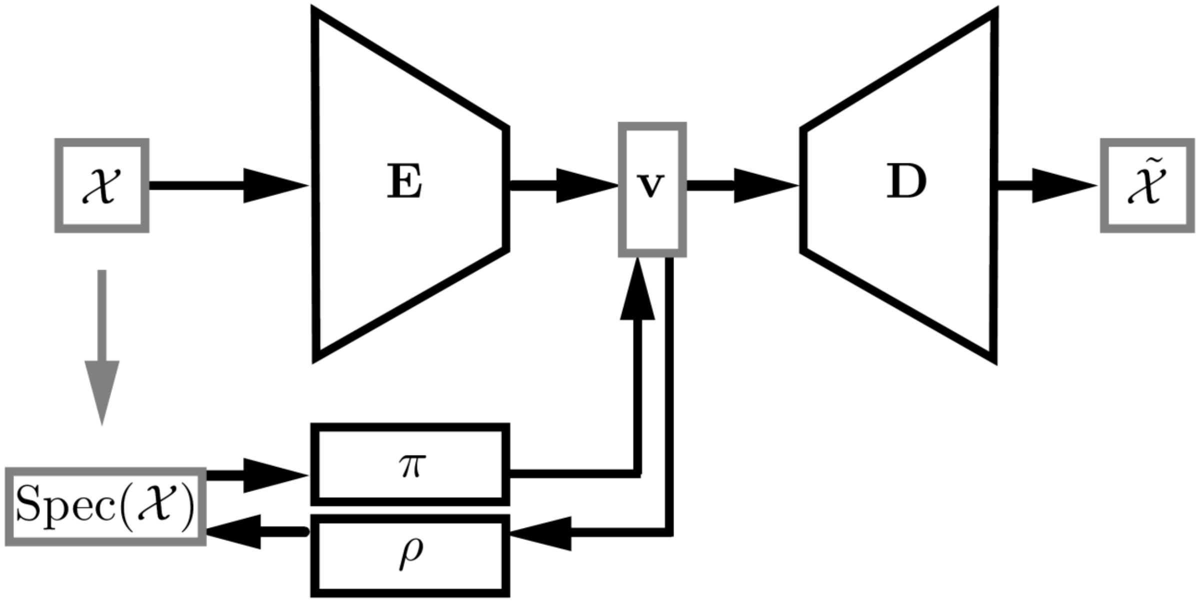


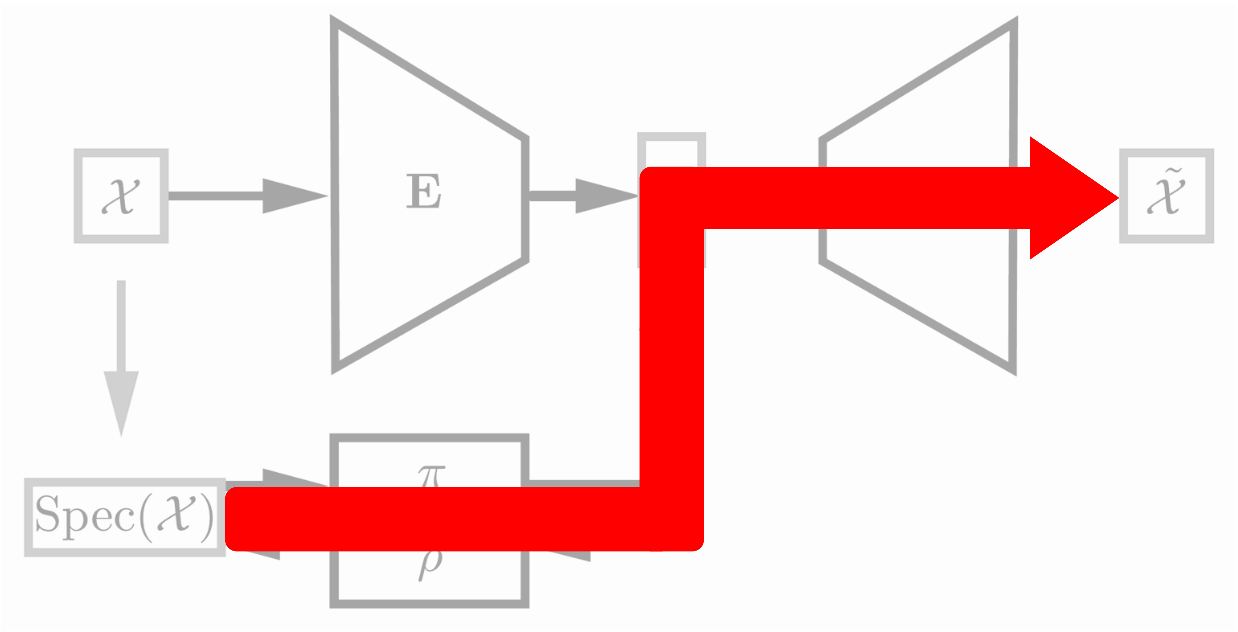
<u>Remarks:</u>

- **No back-propagation** through the eigen-decomposition
- The input spectrum can be **arbitrarily accurate**
- Admits **any AE** model (e.g. for point clouds, meshes, etc.)

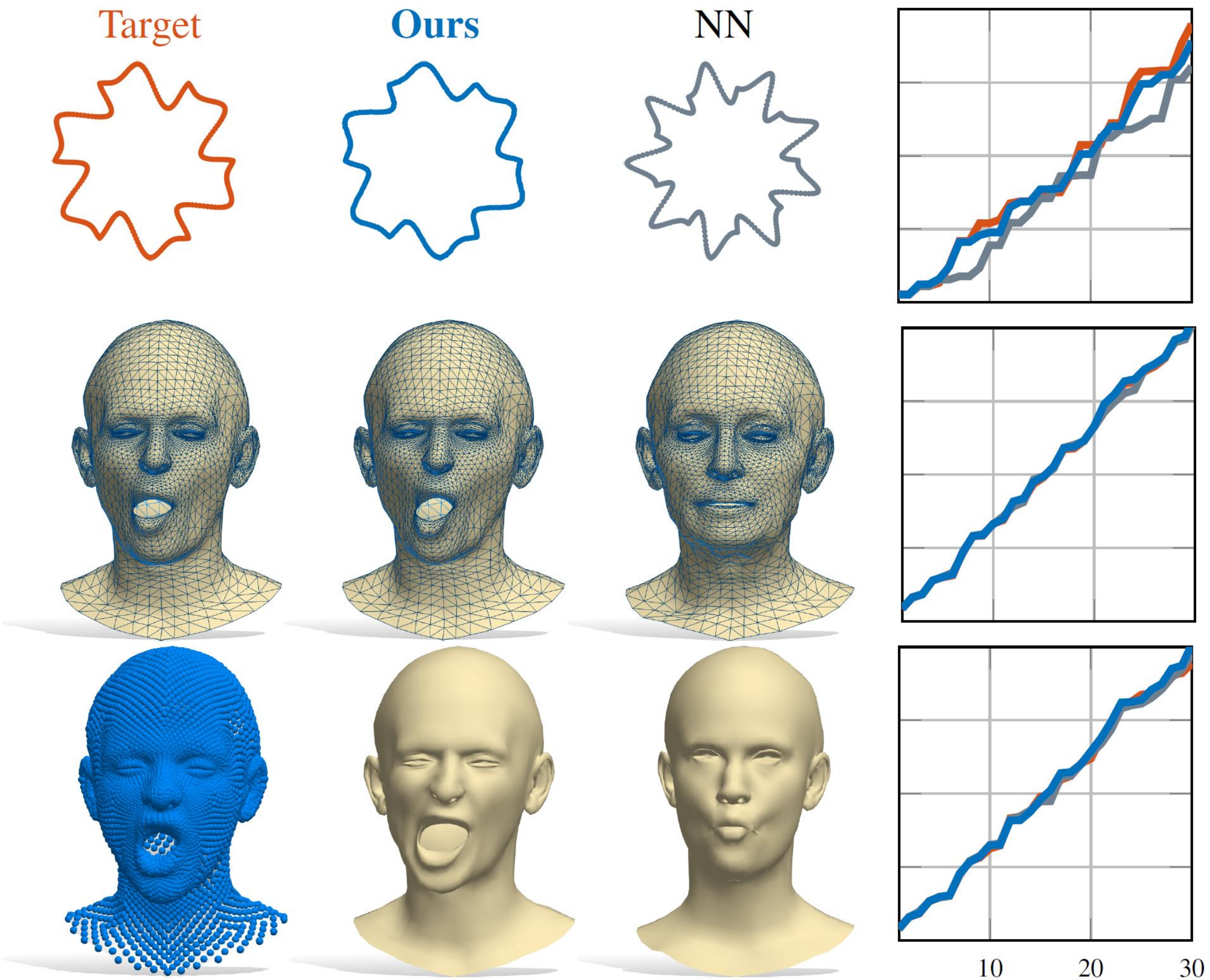
Shape-from-spectrum reconstruction



Shape-from-spectrum reconstruction



Examples



Application: Style transfer

$$\min_{\mathbf{v}} \|\mathrm{Spec}(\mathcal{X}_{\text{style}}) - \rho(\mathbf{v})\|_2^2 + w\|\mathbf{v} - E(\mathcal{X}_{\text{pose}})\|_2^2$$

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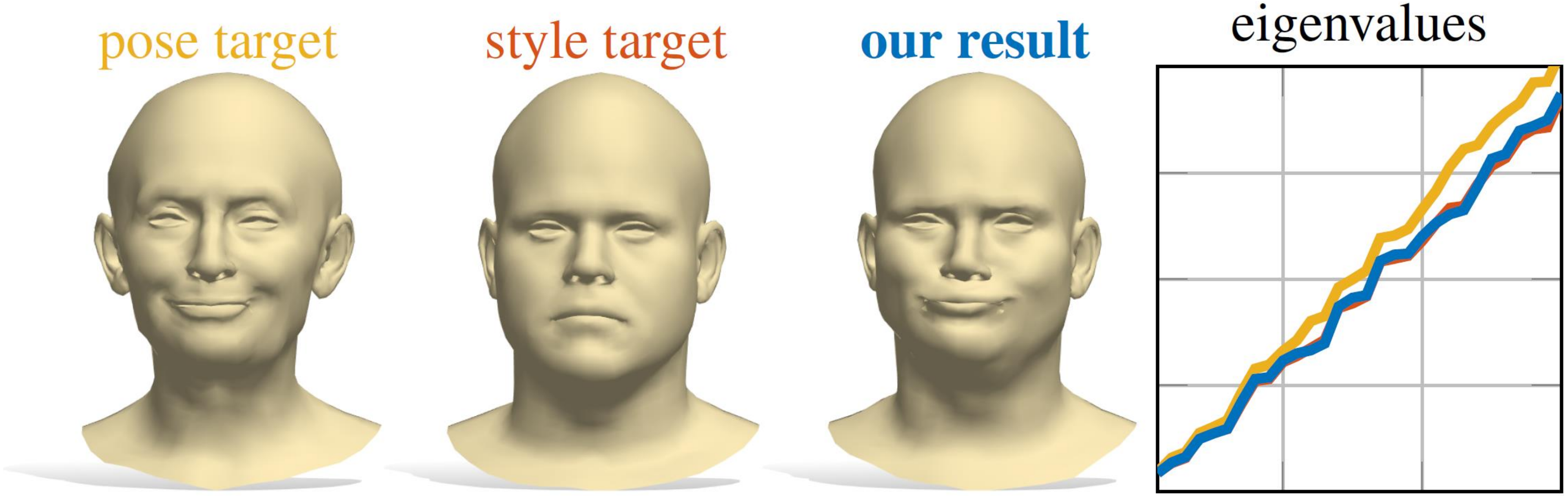
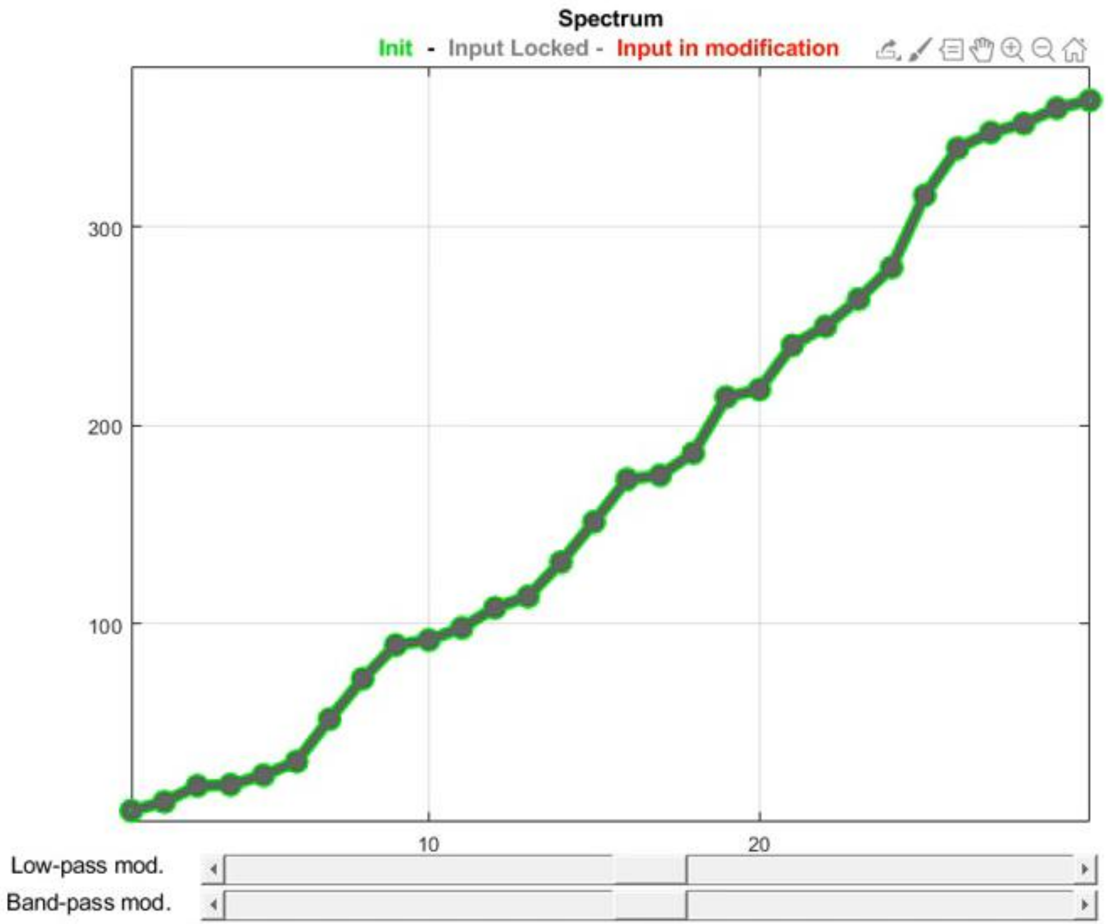
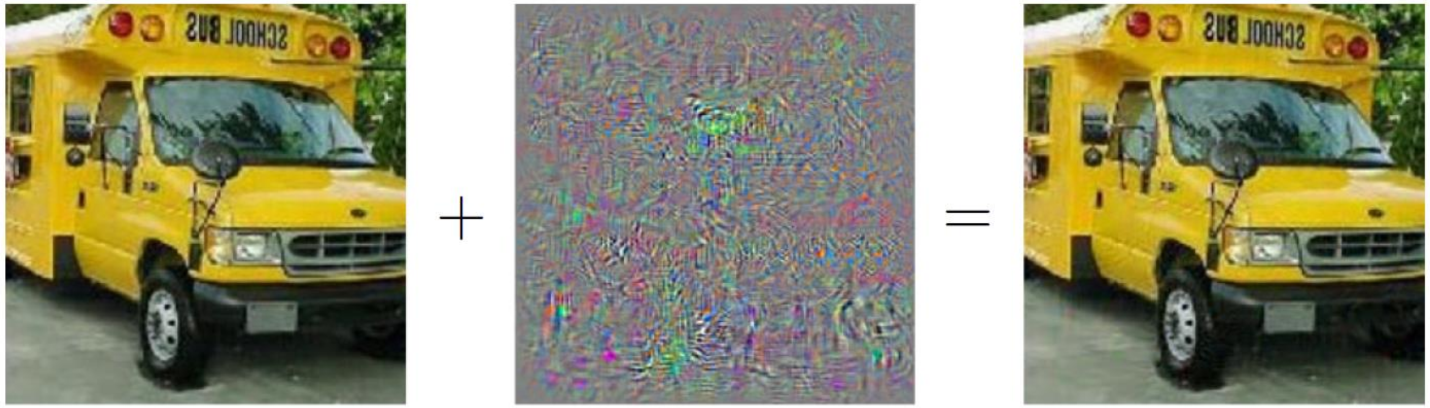


Application: Shape exploration



Adversarial attacks

Adversarial perturbations



“truck” perturbation “ostrich”

The perturbation should be undetectable and can be explicitly optimized for.

Malicious attacks

“speed limit 50mph”

Example of a malicious attack on a visual classifier

Targeted attacks

Given an input sample $\mathbf{x}$, a classifier C, and a target class t, consider:

$$\min_{\mathbf{x}' \in [0,1]^n} \|\mathbf{x} - \mathbf{x}'\|_2^2 \quad \text{s.t. } C(\mathbf{x}') = t$$

We call $\mathbf{x}'$ an adversarial example.

Relax the difficult constraint to a penalty term:

$$\min_{\mathbf{x}' \in [0,1]^n} \|\mathbf{x} - \mathbf{x}'\|_2^2 + c\,L(\mathbf{x}', t)$$

Targeted attacks

A more general approach is given by:

$$\min_{\boldsymbol{\delta} \in [0,1]^n} d(\mathbf{x}, \mathbf{x} + \boldsymbol{\delta}) + c\, f(\mathbf{x} + \boldsymbol{\delta})$$

where the perturbation $\boldsymbol{\delta}$ appears explicitly, and d is some distance

f is such that $C(\mathbf{x} + \boldsymbol{\delta}) = t$ if and only if $f(\mathbf{x} + \boldsymbol{\delta}) \leq 0$.

$$\begin{aligned}
f_1(x') &= -\mathrm{loss}_{F,t}(x') + 1 \\
f_2(x') &= (\max_{i \neq t}(F(x')_i) - F(x')_t)^+ \\
f_3(x') &= \mathrm{softplus}(\max_{i \neq t}(F(x')_i) - F(x')_t) - \log(2) \\
f_4(x') &= (0.5 - F(x')_t)^+ \\
f_5(x') &= -\log(2F(x')_t - 2) \\
f_6(x') &= (\max_{i \neq t}(Z(x')_i) - Z(x')_t)^+ \\
f_7(x') &= \mathrm{softplus}(\max_{i \neq t}(Z(x')_i) - Z(x')_t) - \log(2)
\end{aligned}$$

See:

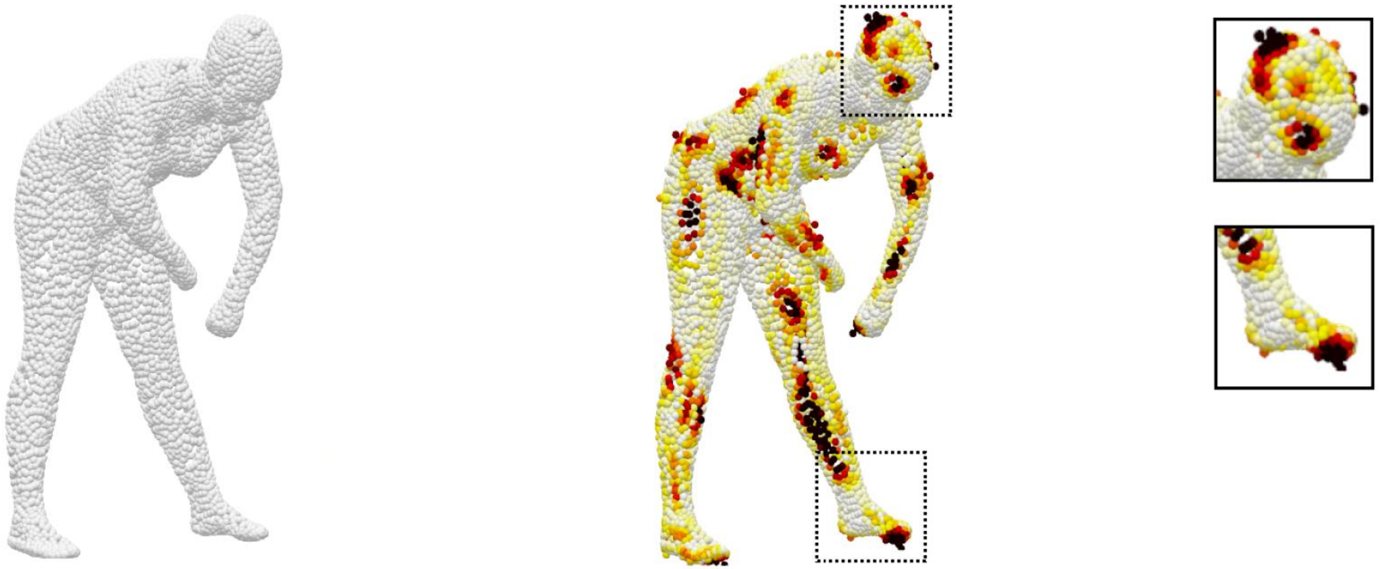
Carlini and Wagner, 2016
“Towards evaluating the robustness of neural networks”

Surface attacks

A perturbation $\mathbf{V}$ is a displacement field:

$$\mathbf{X}' = \mathbf{X} + \mathbf{V}$$

Arbitrary displacement can lead to noticeable adversarial jittering:



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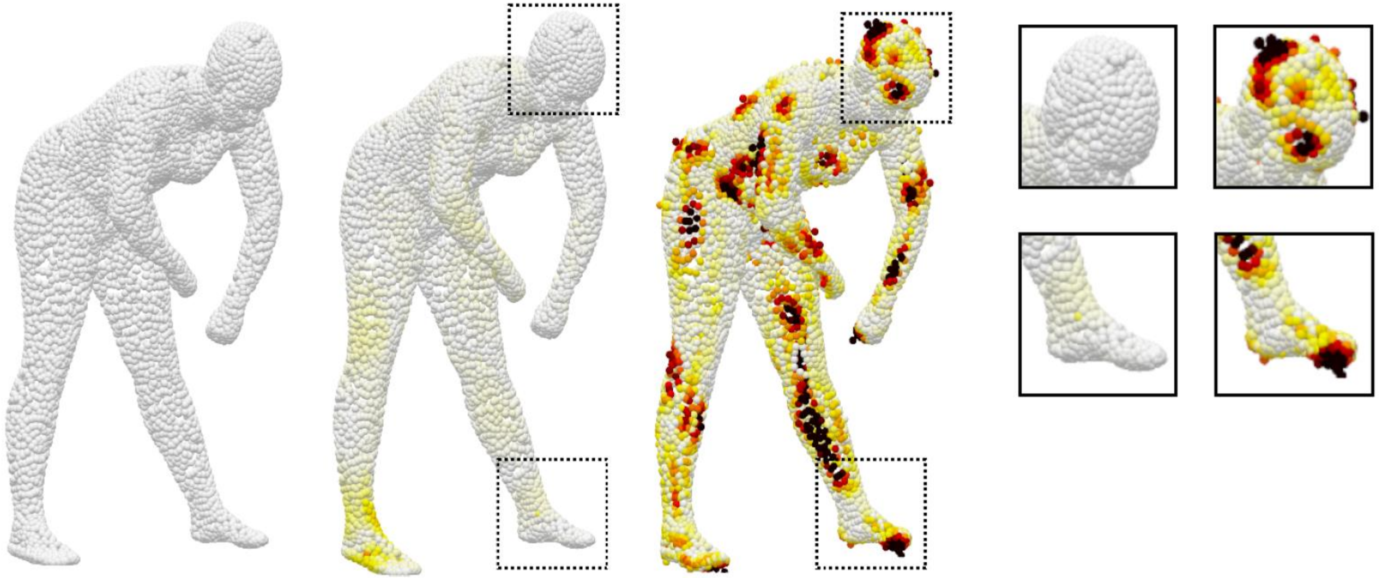

Idea: Regularize the displacement to make it less noticeable.

Band-limited perturbations

Represent the perturbation in the truncated Laplacian eigenbasis:

$$\mathbf{V} = \mathbf{\Phi v}$$

Theorem 1 [ABK15] For any given choice of $k \geq 1$ and any function $f \in \mathcal{F}(\mathcal{X})$, the inequality:

$$\|f - \sum_{i=1}^{k} \langle \psi_i, f \rangle \psi_i\|^2 \leq \alpha \frac{\|\nabla f\|^2}{\lambda_{k+1}} \tag{4}$$

holds for $\alpha = 1$ whenever one chooses ψ_i to be the Laplacian eigenfunctions, while tightening the bound with $0 \leq \alpha < 1$ is not possible for *any* sequence of orthogonal functions $\{\psi_i \in \mathcal{F}(\mathcal{X})\}$.

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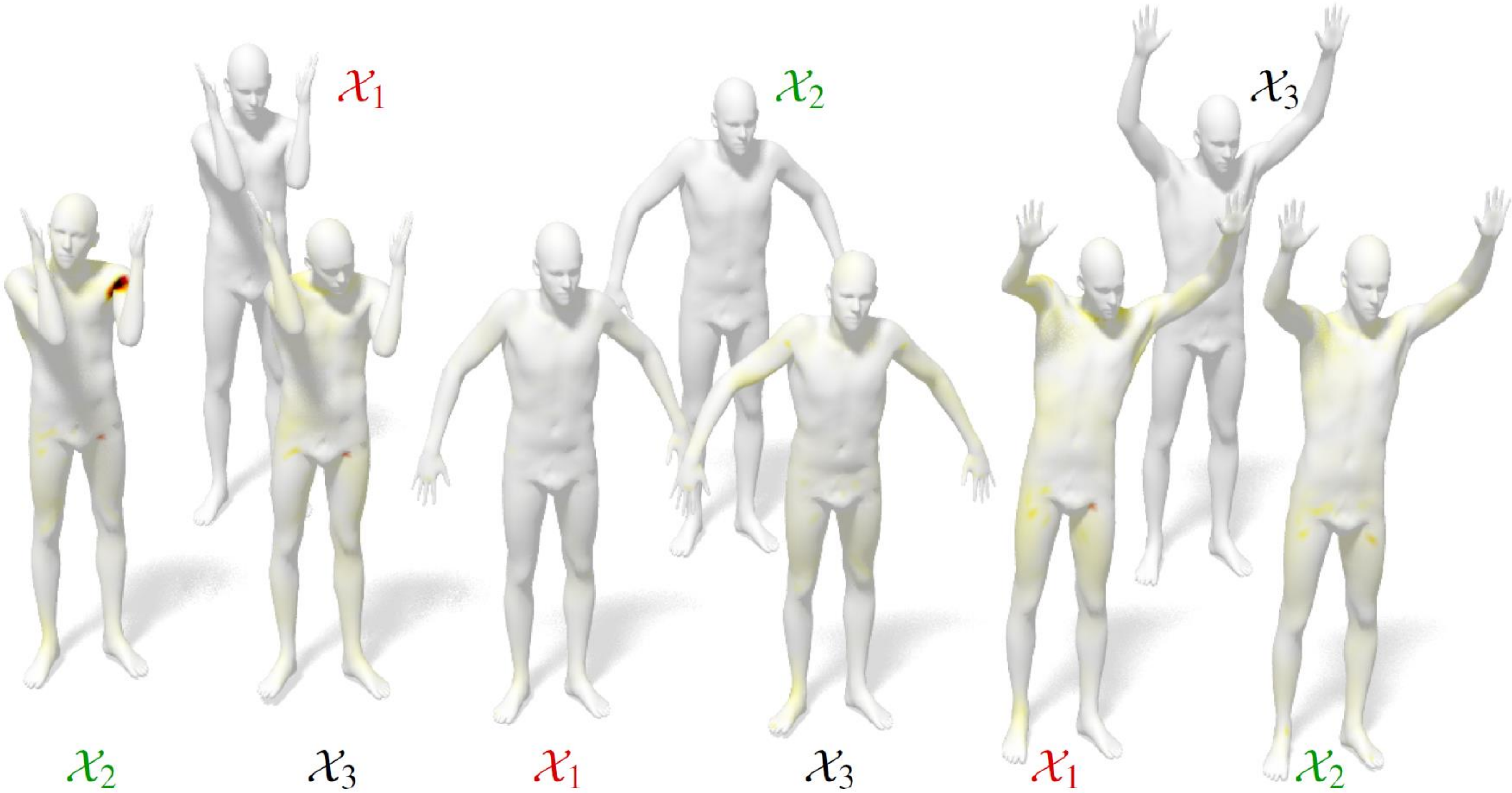
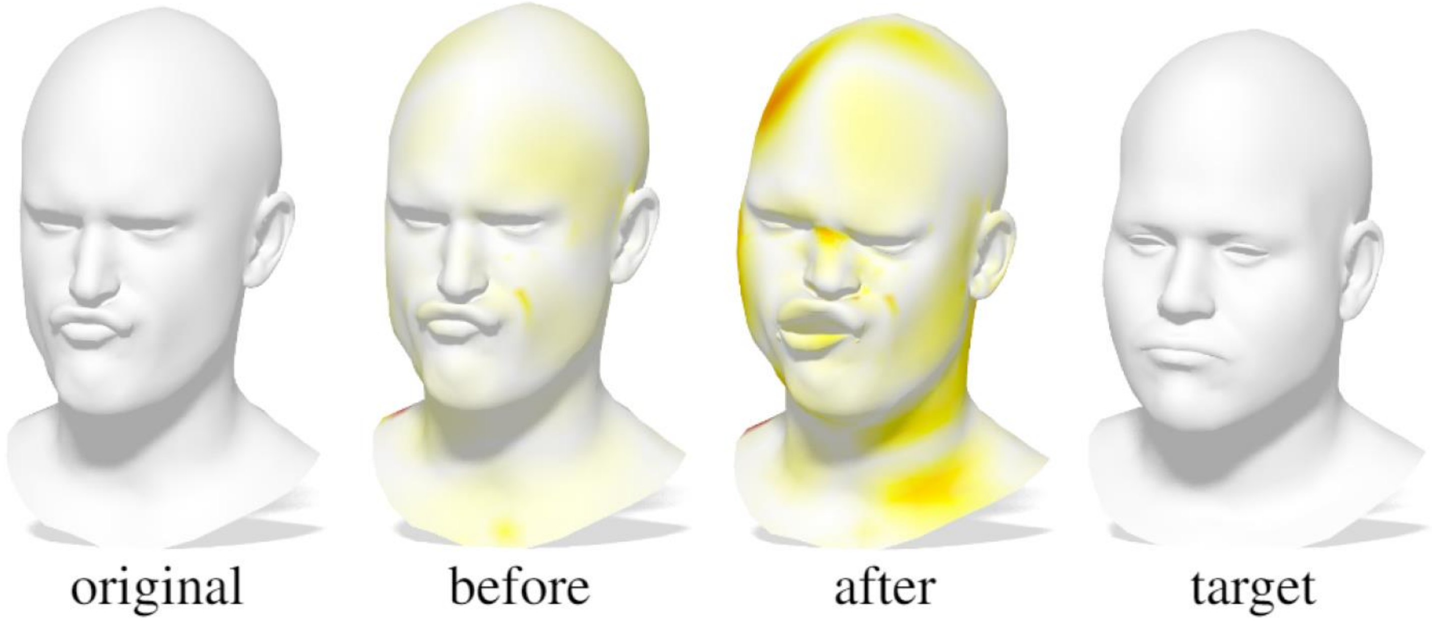
Examples



$\mathcal{X}_1$
$\mathcal{X}_2$
$\mathcal{X}_3$
$\mathcal{X}_2$
$\mathcal{X}_3$
$\mathcal{X}_1$
$\mathcal{X}_3$
$\mathcal{X}_1$
$\mathcal{X}_2$

Adversarial training

Using adversarial examples as training data improves the robustness of the attacked learning model:

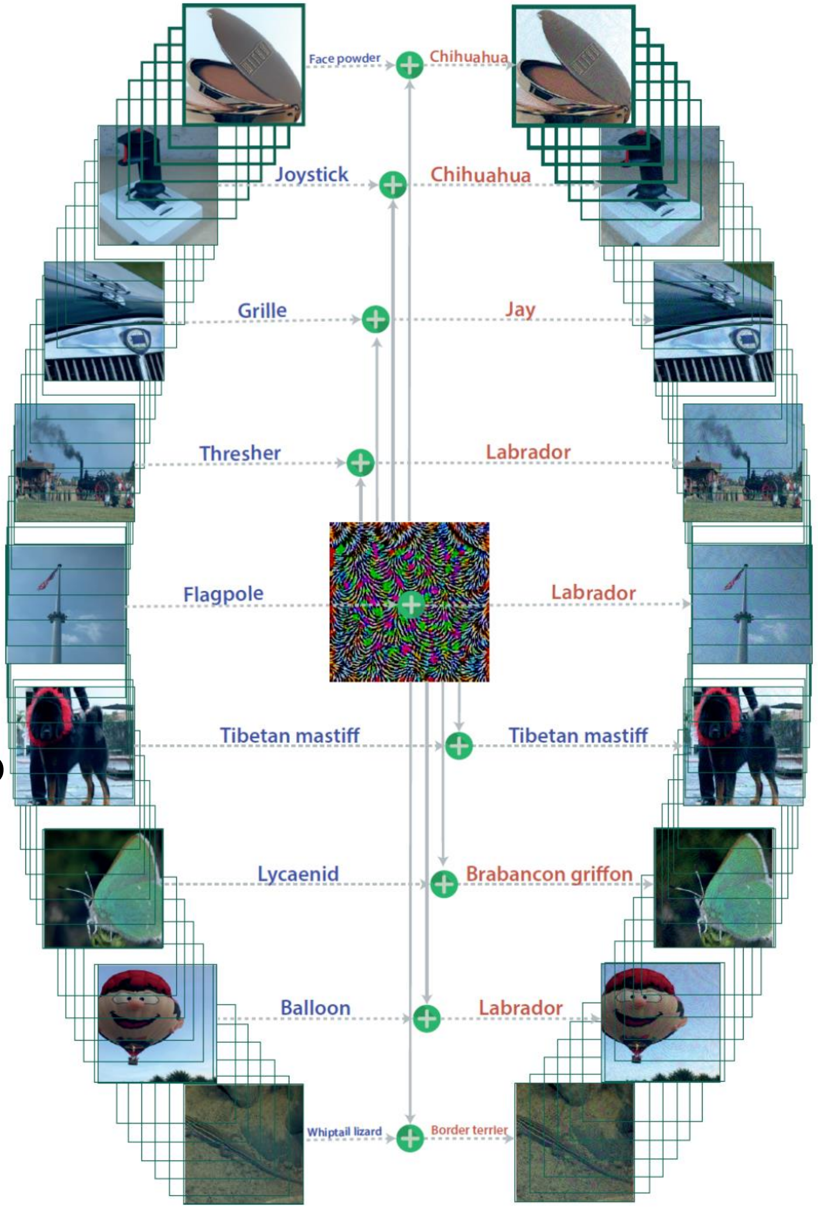


Universal perturbations

Image-agnostic perturbations are known to exist.

What about surfaces and point clouds?

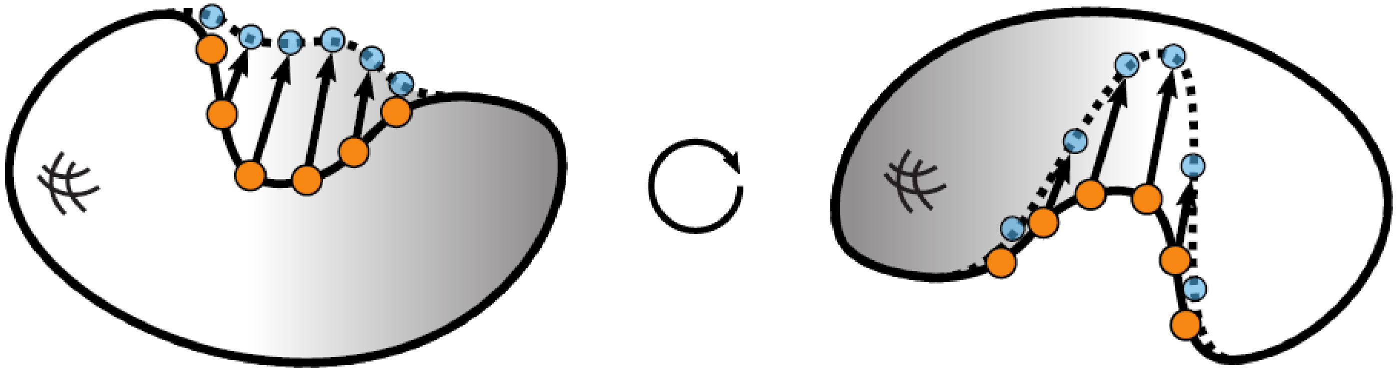
Can we even define a single spatial perturbation for an entire collection of shapes?



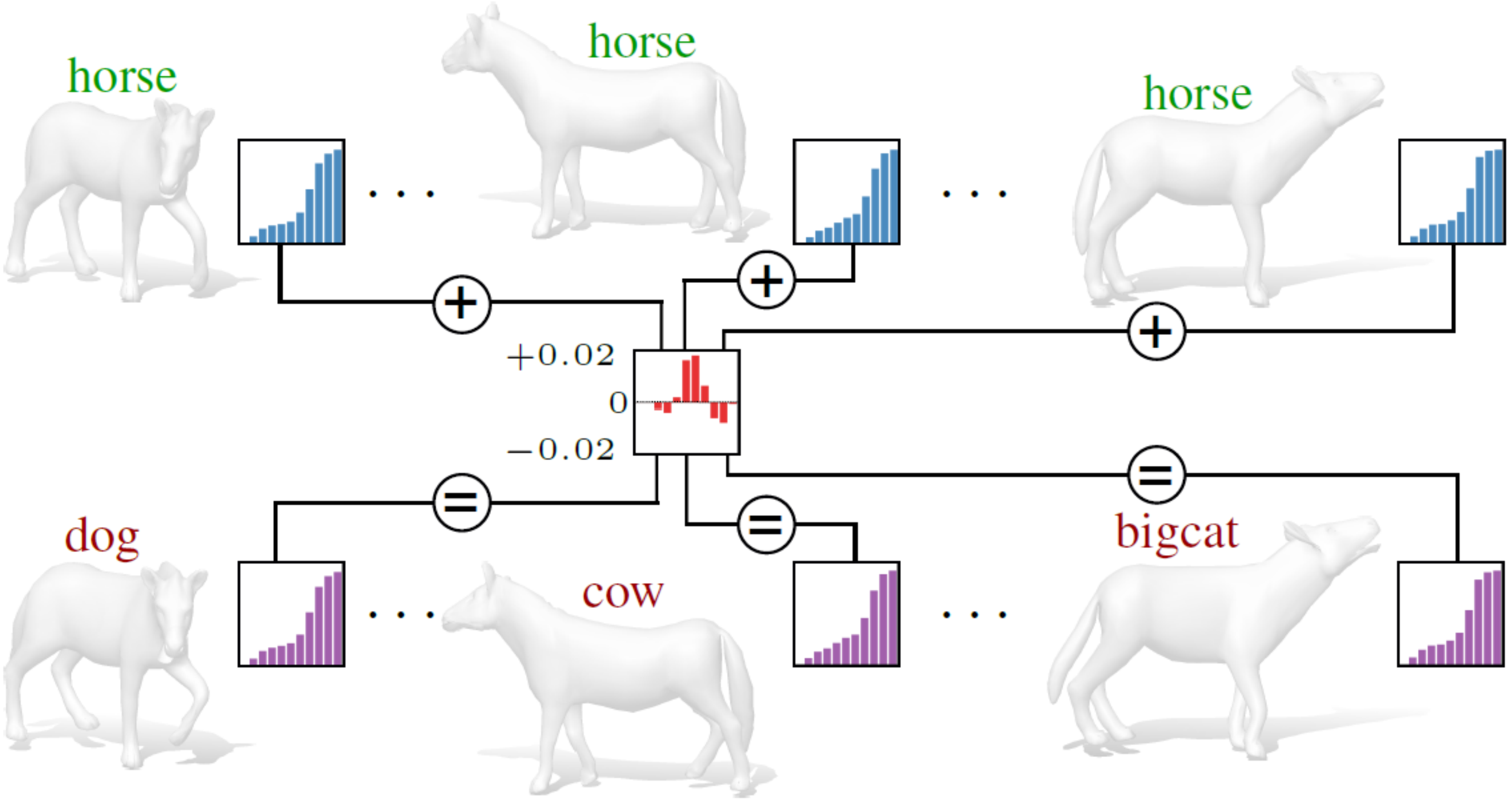
Universal spatial perturbations

No!

Each shape is its own domain.
Any spatial perturbation only applies to the domain where it is defined.



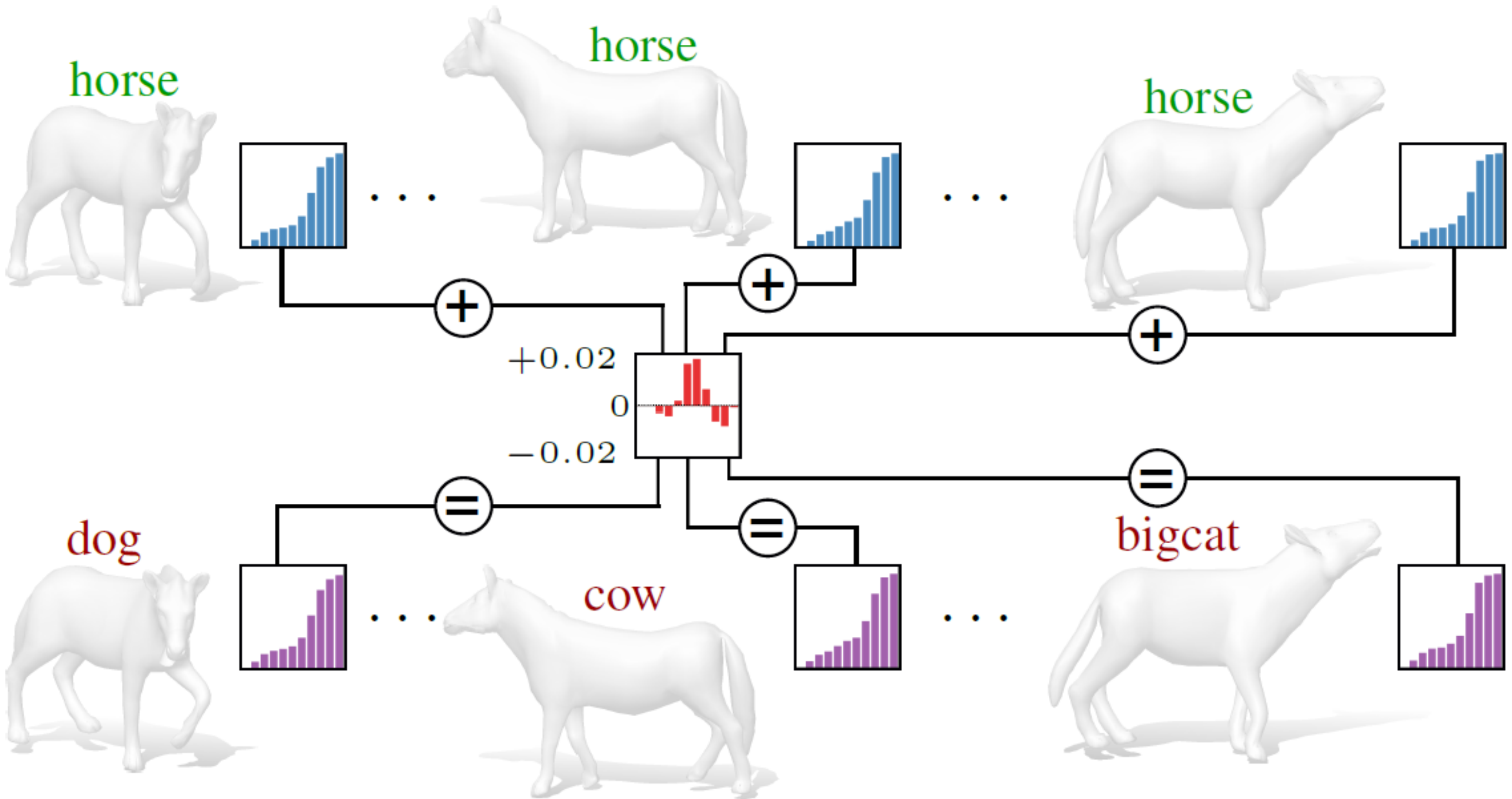
Universal spectral perturbations



Universal spectral perturbations

$$\min_{\substack{\rho\in\mathbb{R}^k\\ \mathcal{P}_i}} \sum_{X_i\in\mathcal{S}} \|\sigma(X_i)(1+\rho)-\sigma(\mathcal{P}_i(X_i))\|_2^2$$

$$\text{s.t. } \mathcal{C}(\mathcal{P}_i(X_i)) \neq \mathcal{C}(X_i) \quad \forall X_i \in \mathcal{S}$$



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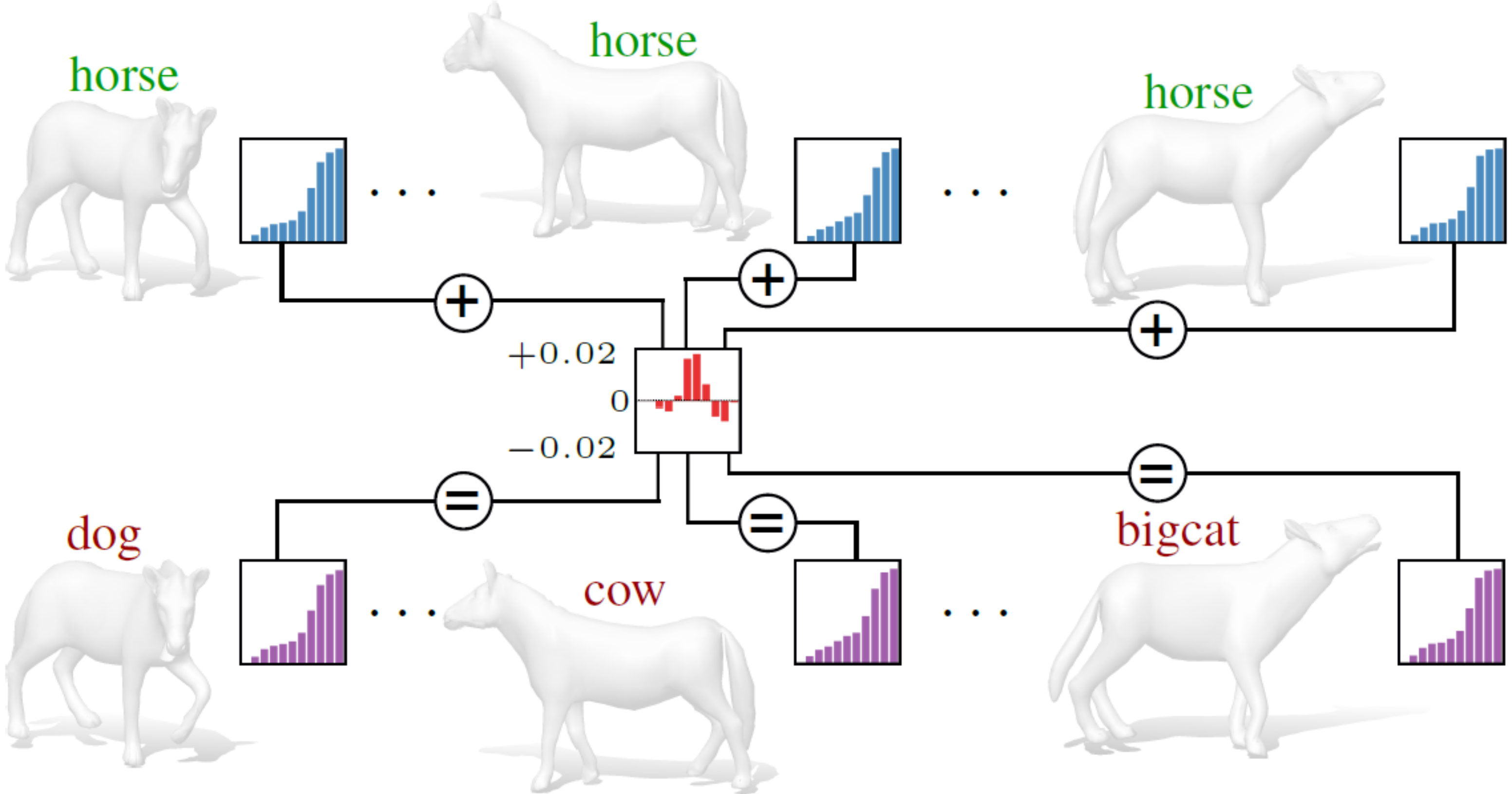
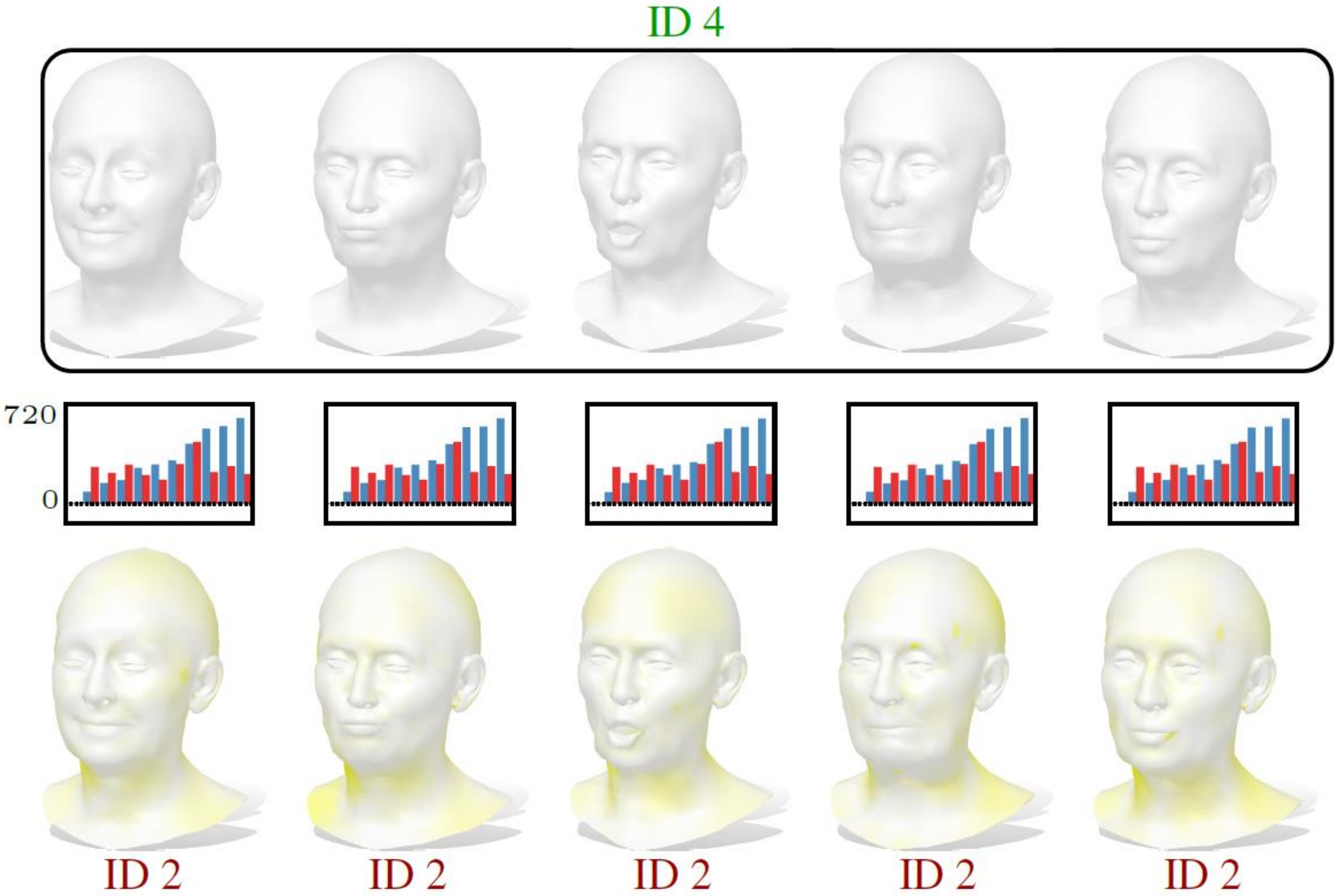
$$\text{s.t. } \mathcal{C}(\mathcal{P}_i(X_i)) \neq \mathcal{C}(X_i) \quad \forall X_i \in \mathcal{S}$$

$$\begin{tikzcd} X_i \arrow[r, "\sigma"] \arrow[d, "\mathcal{P}_i"'] & (\lambda^i) \arrow[d, "\rho"] \\ \tilde{X}_i \arrow[r, "\sigma"] & (\tilde{\lambda}^i) \end{tikzcd}$$



Examples



Thanks for ~~hearing~~ listening!