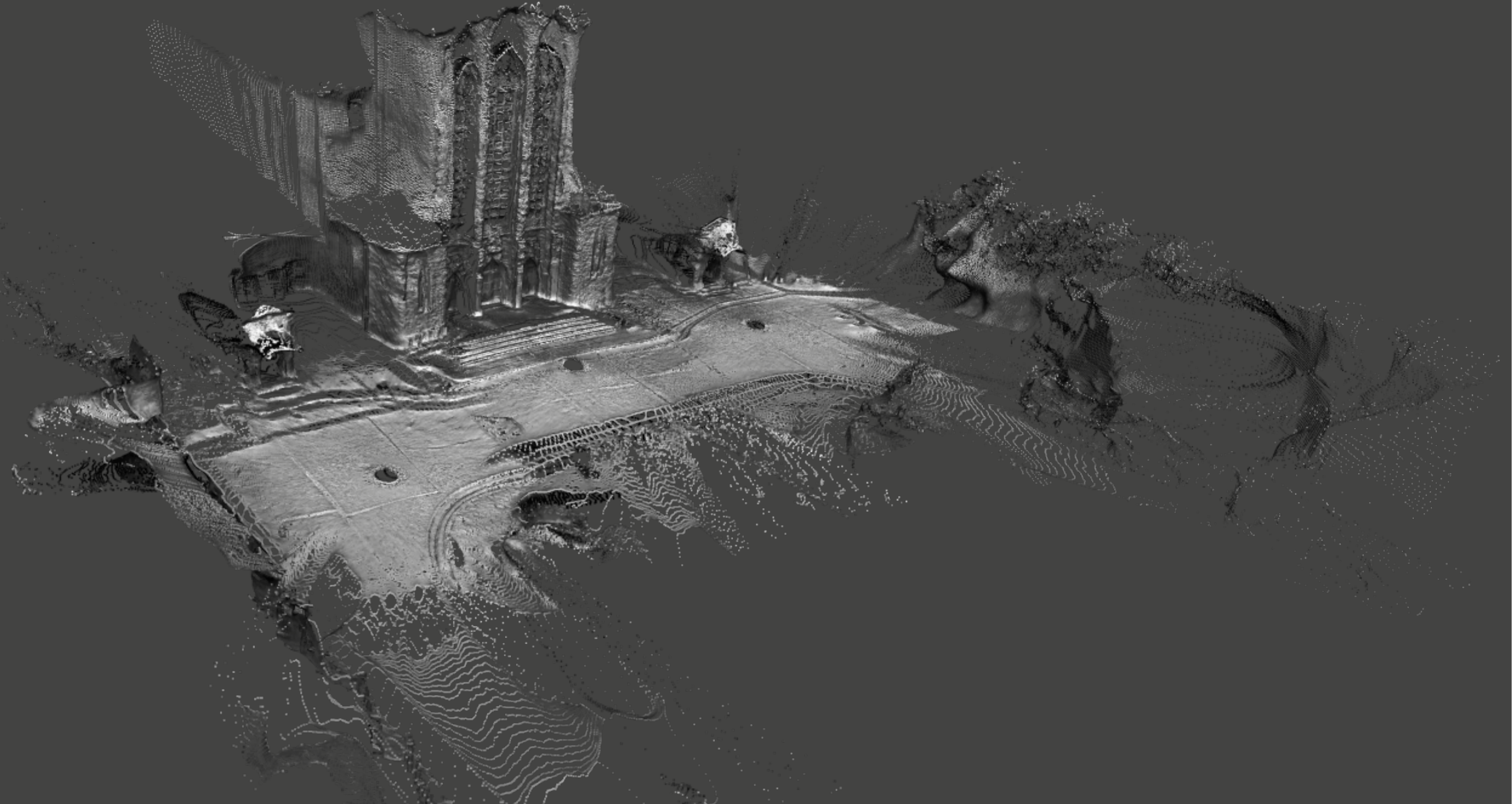


DIFFERENTIAL ANALYSIS OF POINT SET SURFACES AT MULTIPLE SCALES

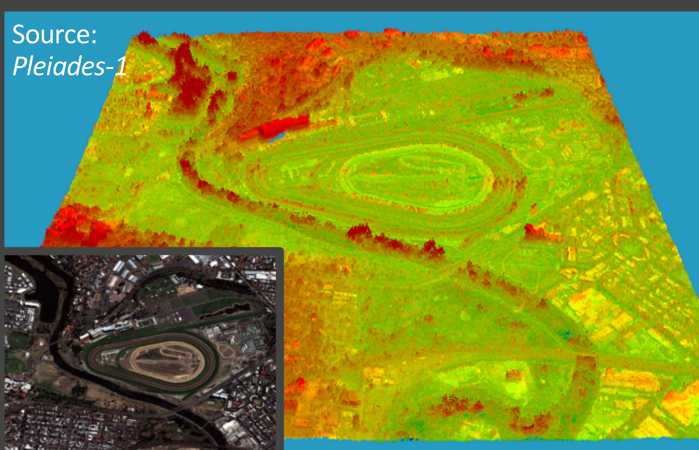
DGDVC
31/03/2021

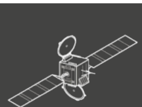
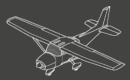
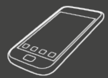
Nicolas Mellado

Point-based shape analysis

Analysis and processing of 3D data

- Motivations
  - 3D surface acquisition techniques are ubiquitous



City  Building  Object 

- Usages
  - Visualization, metrology, simulation, fabrication, ...

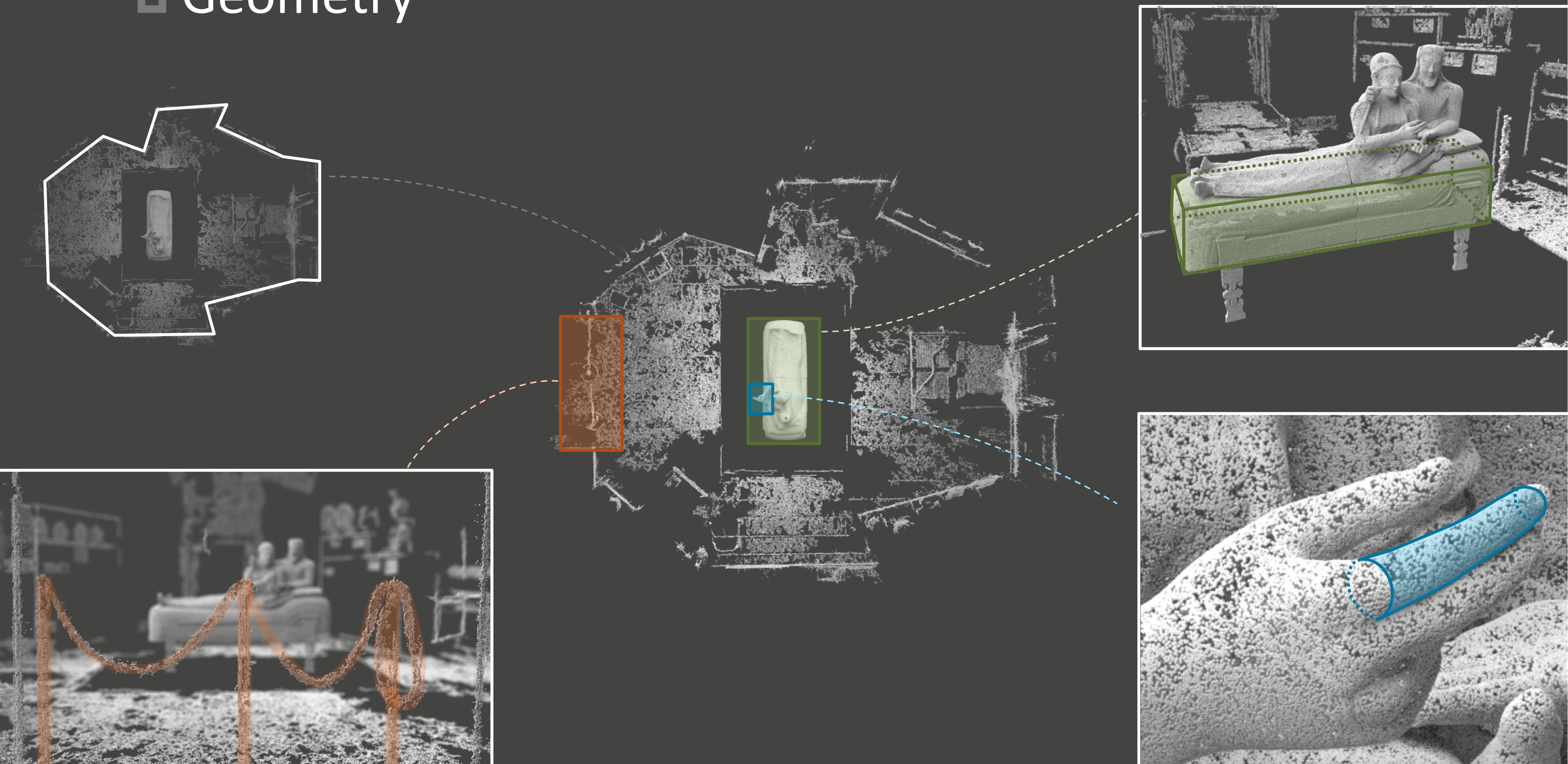
Analysis and processing of 3D data

- Motivations
  - Acquired 3D surfaces are complex
    - Complex geometry, multiple scales (far from base/relief)
    - Massive amount of data (billions of points)

Subsampled: 500M
Original: 20 billions

Objectives

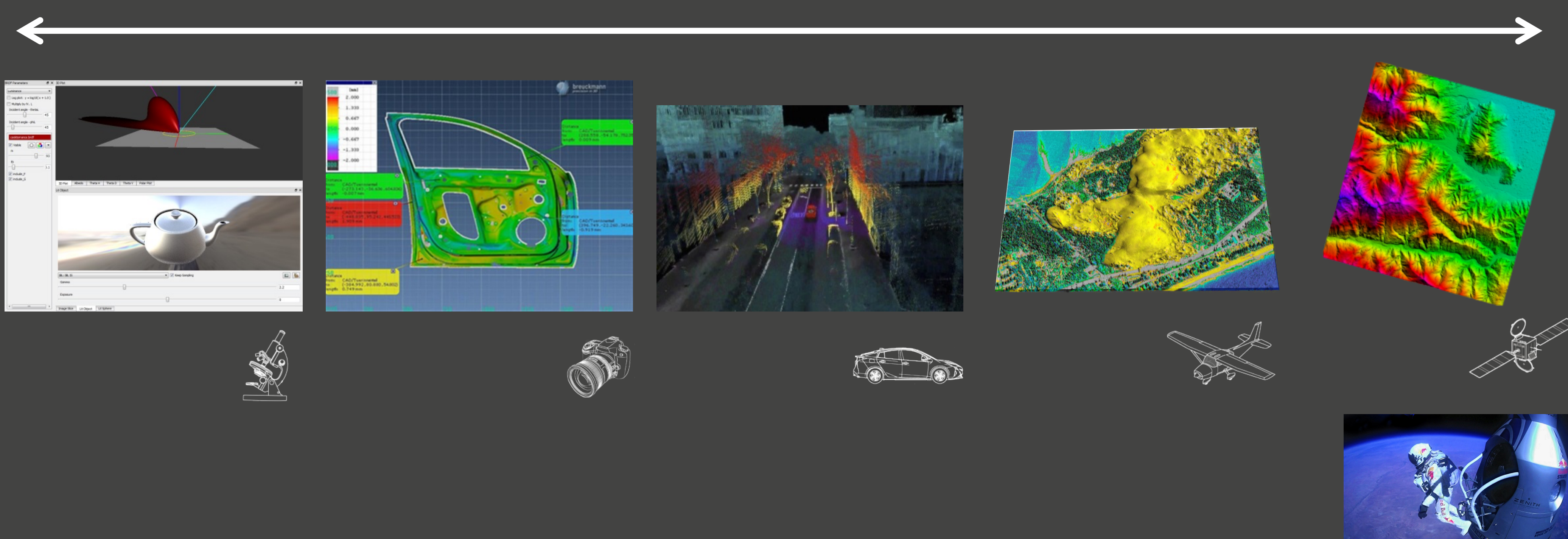
- Characterizing and structuring raw data
  - Geometry



Objectives

- Characterizing and structuring raw data
  - Geometry
  - Multi-scale data continuum

Continuum

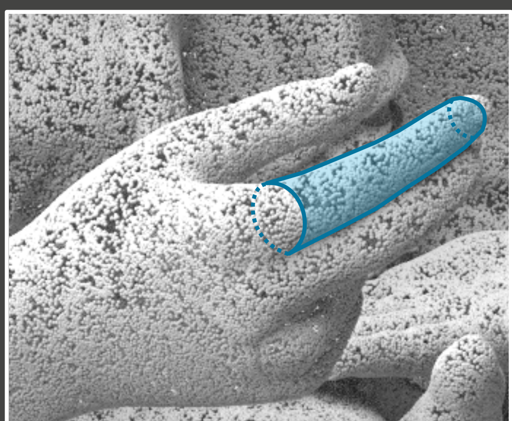


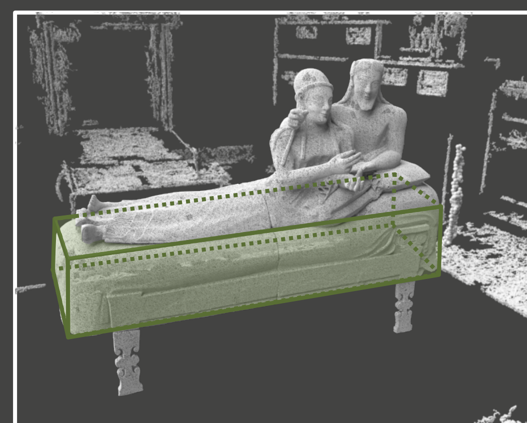
Objectives

- Characterizing and structuring raw data
  - Geometry
  - Multi-scale data continuum
  - Interactive processing
    - Keep user in the loop
      - What is simple for users can be complex for computers
    - Characterizing properties of any sample
      - Users want to augment the data
    - Characterizing properties at any scale
      - Relevant structures can be at any scale
    - Be efficient
      - Robustness to acquisition artefacts and complexity

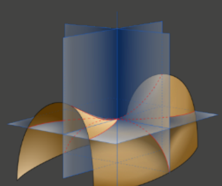
Challenges

- Limited expressiveness of analysis techniques

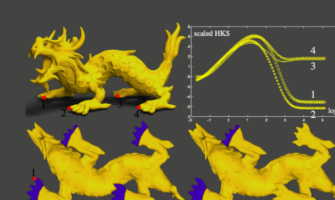




Local
- differential geometry

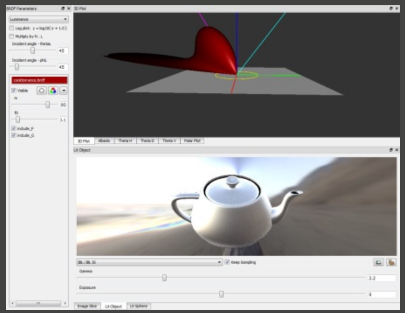


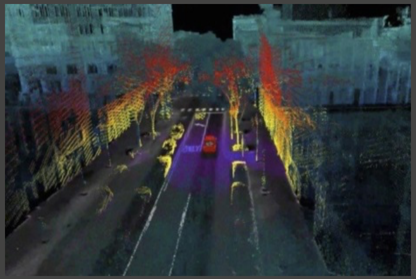
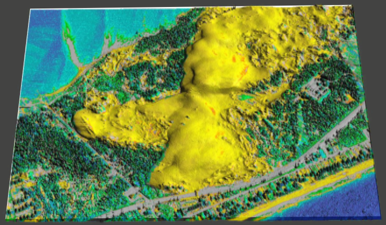
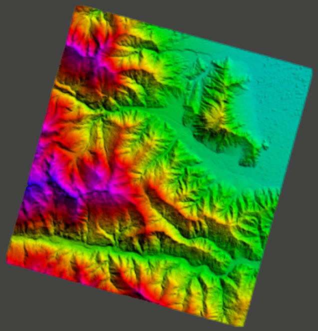
Global
- harmonics
- diffusion



Challenges

- Limited expressiveness of analysis techniques
- Definition of *pertinent* scale/structure



Scale continuum

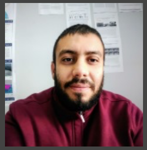
Question: how do we split this continuum, at which scales ?
Which shapes shall we consider ?

Research group

- Nicolas Mellado - Point based analysis and processing
- Loic Barthe - implicit representations, 3D modelling

Task force

- Thibault Lejemble - PhD Student (2017-2020)
- Chems-Eddine Himeur - PhD Student (2021-2024)
- Sébastien EGNER - Master 2 Intern (2021)



Data provider and final user

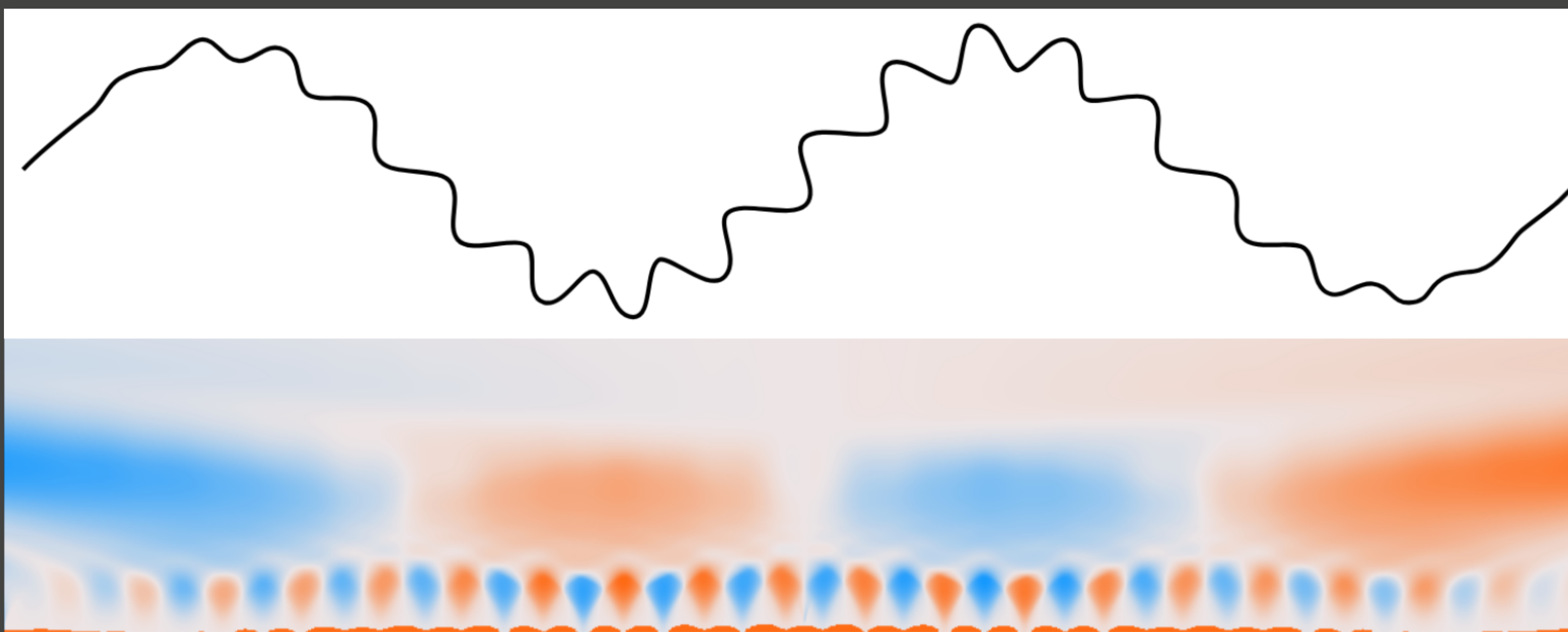
- UMS ArcheoVision

Lalibela Churches
Ethiopy

Open-Science and tools

- Open-source libraries
  - Point cloud analysis https://github.com/poncateam/ponca
  - 3D-Engine https://github.com/STORM-IRIT/Radium-Engine
  - Point-based Deep-Learning library (not released yet)
- Datasets
  - 3D-Acquired Research Dataset https://3dard.cnrs.fr
- Acquisition devices
  - RGB-D Camera: Kinect v2
  - Solid-State Lidar: Intel L515, Ipad Pro

Implicit Scale-Space



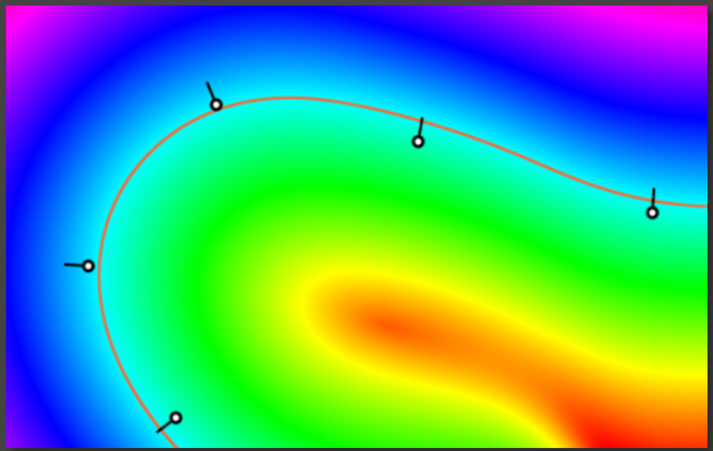
Implicit Scale-Space

- Overall idea
  - See points as surface samples
  - Estimate and study the surface properties
- Proposal
  - Use *implicit* surface reconstruction
  - Study differential properties
    - In space
    - In scale
    - …

Technical background

- Implicit surfaces
  - Isosurface of a scalar field $S_{\mathbf{u}}$

$$(\mathbf{x}; S_{\mathbf{u}}(\mathbf{x}) = 0)$$



Technical background

- Implicit surfaces
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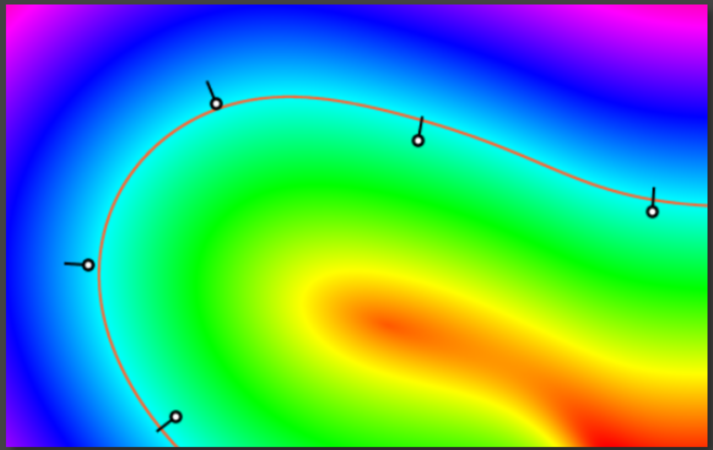
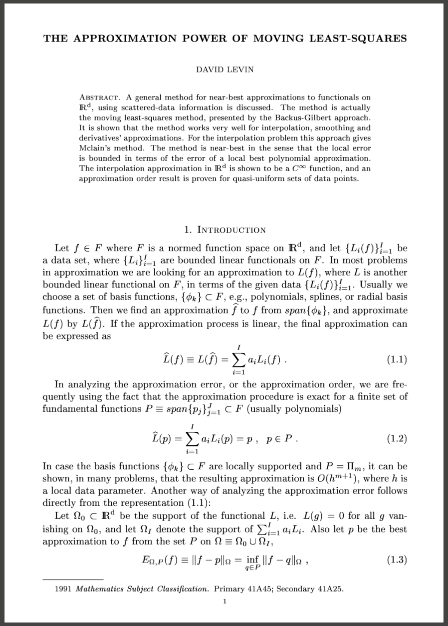


- Pro
  - Derivatives are meaningful, eg., $\nabla S_{\mathbf{u}} \sim$ normal vector
  - Projection
- Cons
  - No explicit definition -> marching cubes
  - No surface metric -> local estimation knn graph

Technical background

- Implicit surface reconstruction
  - Moving Least Squares



THE APPROXIMATION POWER OF MOVING LEAST-SQUARES

DAVID LEVIN

Abstract. A general method for near-best approximations to functionals on $\mathbb{R}^d$, using scattered-data information is discussed. The method is actually the moving least-squares method, presented by the Backus-Gilbert approach. It is shown that the method works very well for interpolation, smoothing and derivatives' approximations. For the interpolation problem this approach gives Mclain's method. The method is near-best in the sense that the local error is bounded in terms of the error of a local best polynomial approximation. The interpolation approximation in $\mathbb{R}^d$ is shown to be a C^∞ function, and an approximation order result is proven for quasi-uniform sets of data points.

1. Introduction

Let $f \in F$ where F is a normed function space on $\mathbb{R}^d$, and let $\{L_i(f)\}_{i=1}^I$ be a data set, where $\{L_i\}_{i=1}^I$ are bounded linear functionals on F. In most problems in approximation we are looking for an approximation to $L(f)$, where L is another bounded linear functional on F, in terms of the given data $\{L_i(f)\}_{i=1}^I$. Usually we choose a set of basis functions, $\{\phi_k\} \subset F$, e.g., polynomials, splines, or radial basis functions. Then we find an approximation $\hat{f}$ to f from $span\{\phi_k\}$, and approximate $L(f)$ by $L(\hat{f})$. If the approximation process is linear, the final approximation can be expressed as

$$\hat{L}(f) \equiv L(\hat{f}) = \sum_{i=1}^{I} a_i L_i(f) . \quad (1.1)$$

In analyzing the approximation error, or the approximation order, we are frequently using the fact that the approximation procedure is exact for a finite set of fundamental functions $P \equiv span\{p_j\}_{j=1}^J \subset F$ (usually polynomials)

$$\hat{L}(p) = \sum_{i=1}^{I} a_i L_i(p) = p , \quad p \in P . \quad (1.2)$$

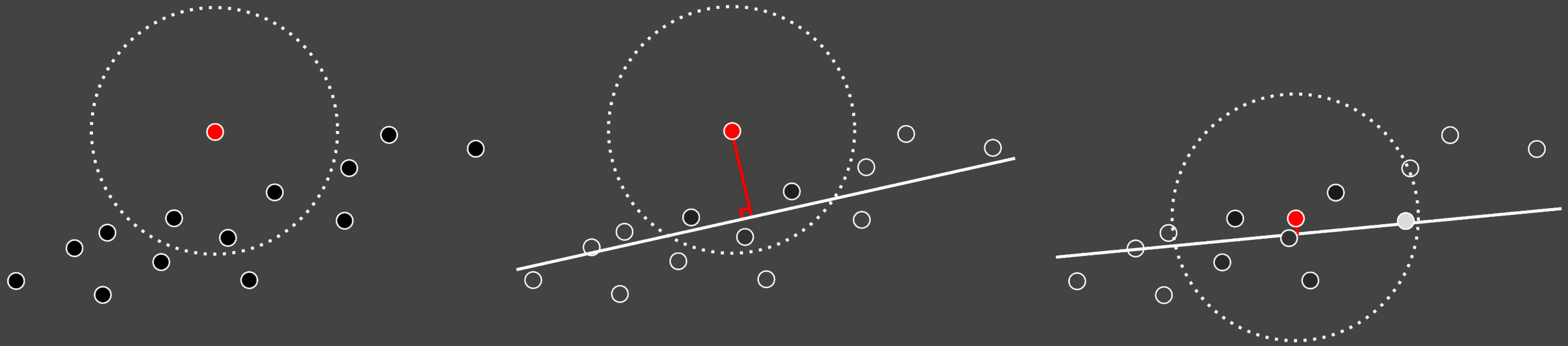
In case the basis functions $\{\phi_k\} \subset F$ are locally supported and $P = \Pi_m$, it can be shown, in many problems, that the resulting approximation is $O(h^{m+1})$, where h is a local data parameter. Another way of analyzing the approximation error follows directly from the representation (1.1):

Let $\Omega_0 \subset \mathbb{R}^d$ be the support of the functional L, i.e. $L(g) = 0$ for all g vanishing on Ω_0, and let Ω_I denote the support of $\sum_{i=1}^I a_i L_i$. Also let p be the best approximation to f from the set P on $\Omega \equiv \Omega_0 \cup \Omega_I$,

$$E_{\Omega,P}(f) \equiv \|f - p\|_\Omega = \inf_{q \in P} \|f - q\|_\Omega , \quad (1.3)$$

1991 *Mathematics Subject Classification.* Primary 41A45; Secondary 41A25.

1



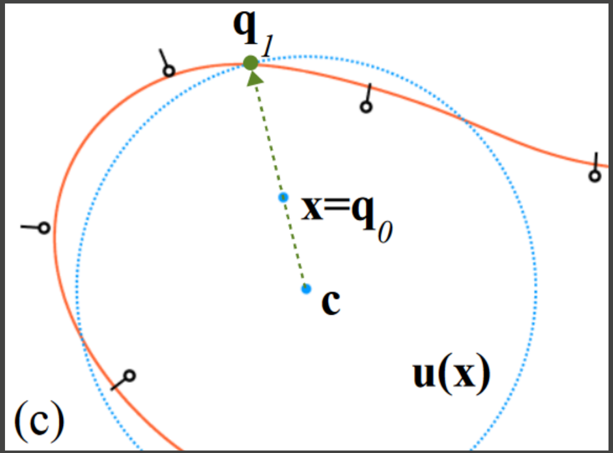
Sources: https://nccastaff.bournemouth.ac.uk/rsouthern/researchblog/mls/

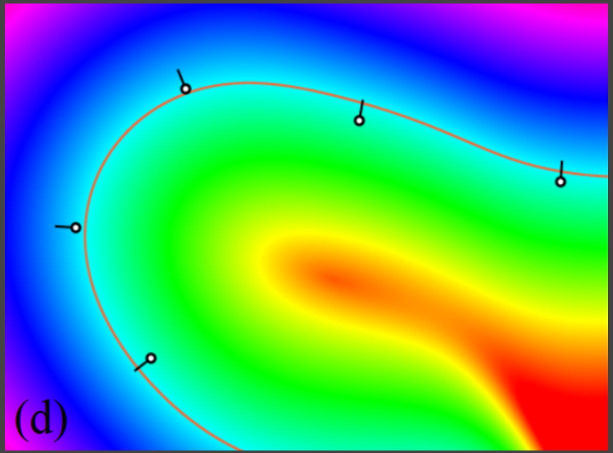
Technical background

- Implicit surface reconstruction
  - Moving Least Squares
  - Algebraic Point Set Surfaces

$$\arg\min_{S_{\mathbf{u}}} \sum_i w_i(t) \| \nabla S_{\mathbf{u}}(\mathbf{q}_i) - \mathbf{n}_i \|^2$$

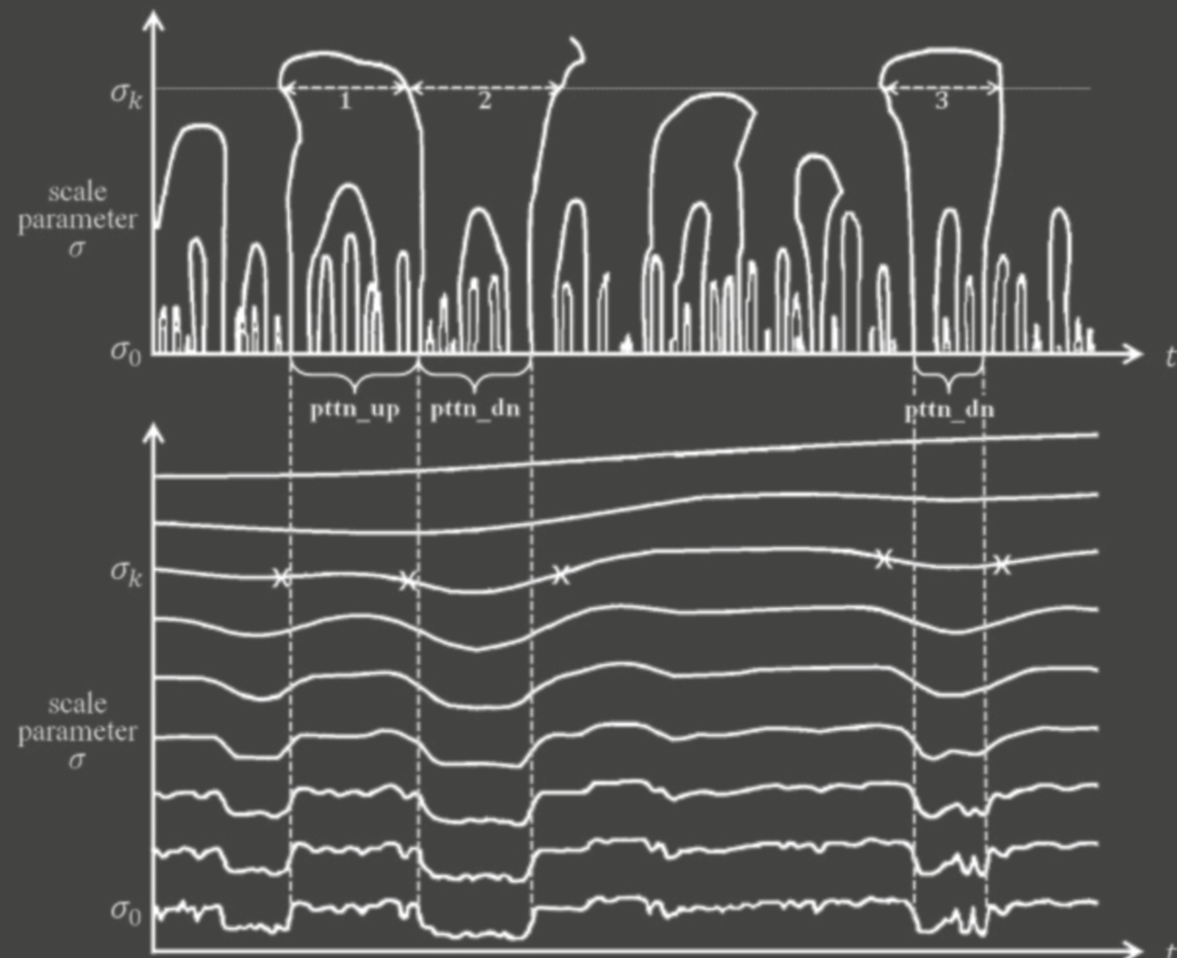
$$S_{\mathbf{u}}(\mathbf{x}) = \mathbf{u}^{\mathrm{T}} \begin{bmatrix} 1 \\ \mathbf{x} \\ \mathbf{x}^{\mathrm{T}}\mathbf{x} \end{bmatrix}, \text{ with } \mathbf{u} = \begin{bmatrix} u_c \\ \mathbf{u}_n \\ u_q \end{bmatrix}$$





Technical background

- Scale-Space
  - Sparse analysis
  - Built on signal parameterization

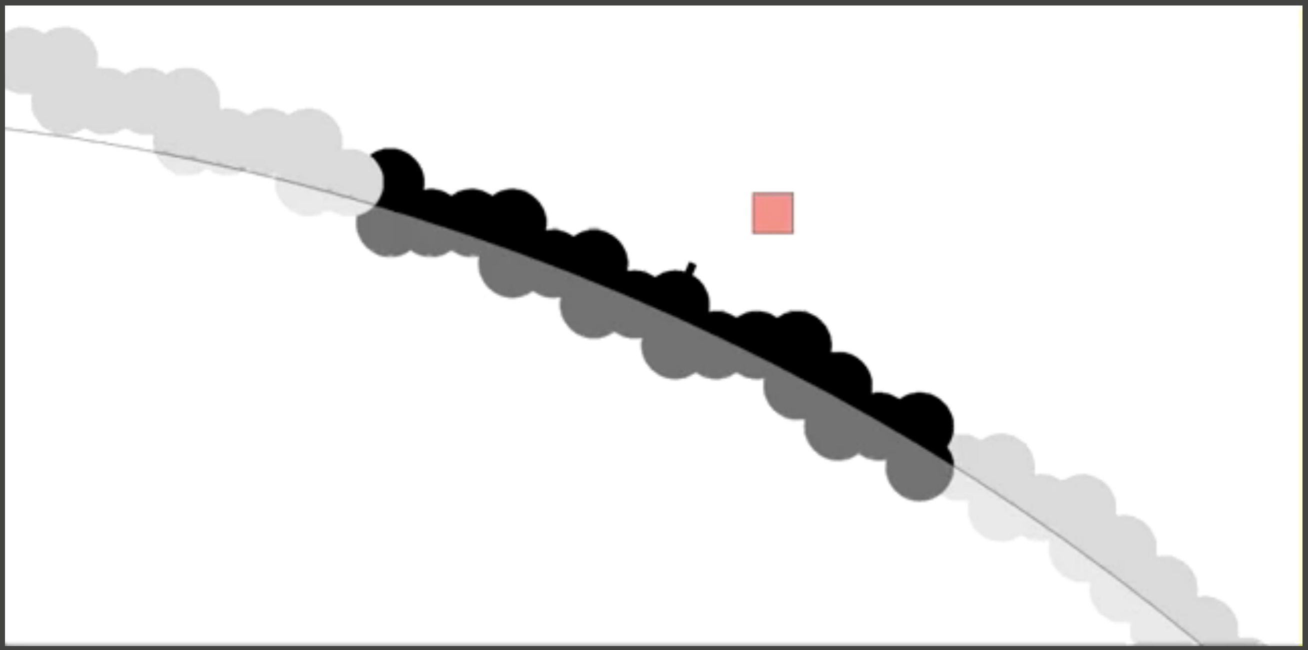


A computational method for detecting copy number variations using scale-space filtering.
Lee J, Lee U, Kim B, Yoon J. BMC Bioinformatics. 2013 Feb 18;14:57. doi: 10.1186/1471-2105-14-57.

Local analysis: pertinent scale extraction

- Proposal: measure pertinence as stability in scale-space

$$S_{\mathbf{u}}(\mathbf{x}) = \mathbf{u}^{\mathrm{T}} \begin{bmatrix} 1 \\ \mathbf{x} \\ \mathbf{x}^{\mathrm{T}}\mathbf{x} \end{bmatrix}, \text{ with } \mathbf{u} = \begin{bmatrix} u_c \\ \mathbf{u}_n \\ u_q \end{bmatrix} \longrightarrow \begin{bmatrix} \tau \\ \boldsymbol{\eta} \\ \kappa \end{bmatrix}$$

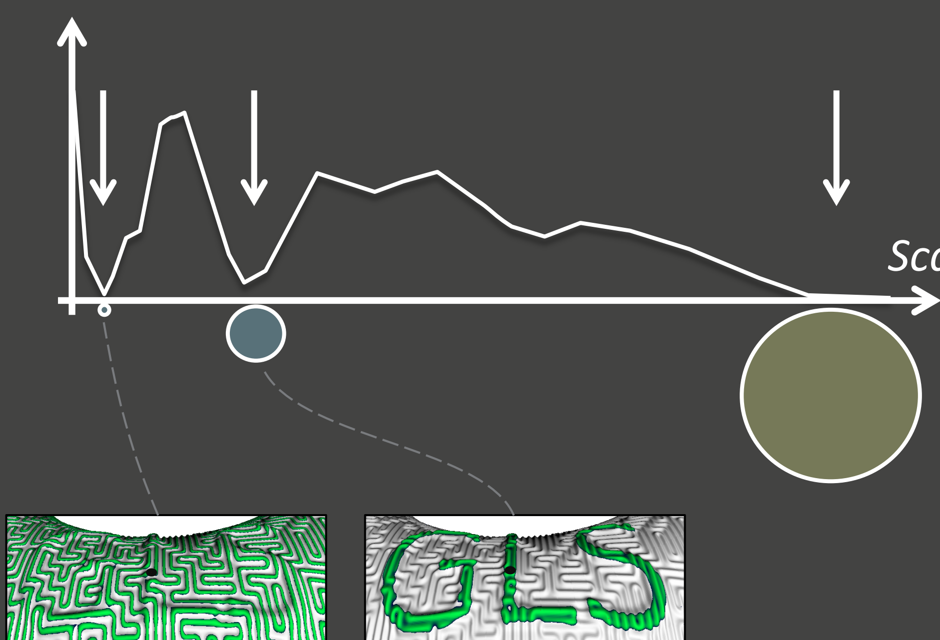


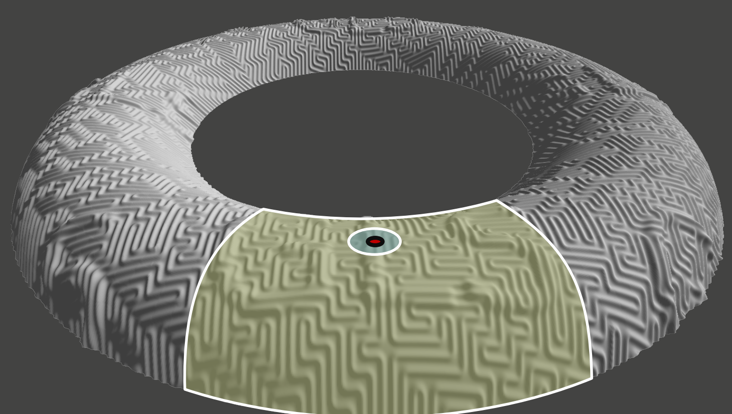
[CGF'12]

Local analysis: pertinent scale extraction

- Proposal: measure pertinence as stability in scale-space

$$pertinence(t) = w_\tau \frac{\delta\tau}{\delta t} + w_{\boldsymbol{\eta}} \frac{\delta\boldsymbol{\eta}}{\delta t} + w_\kappa \frac{\delta\kappa}{\delta t}$$

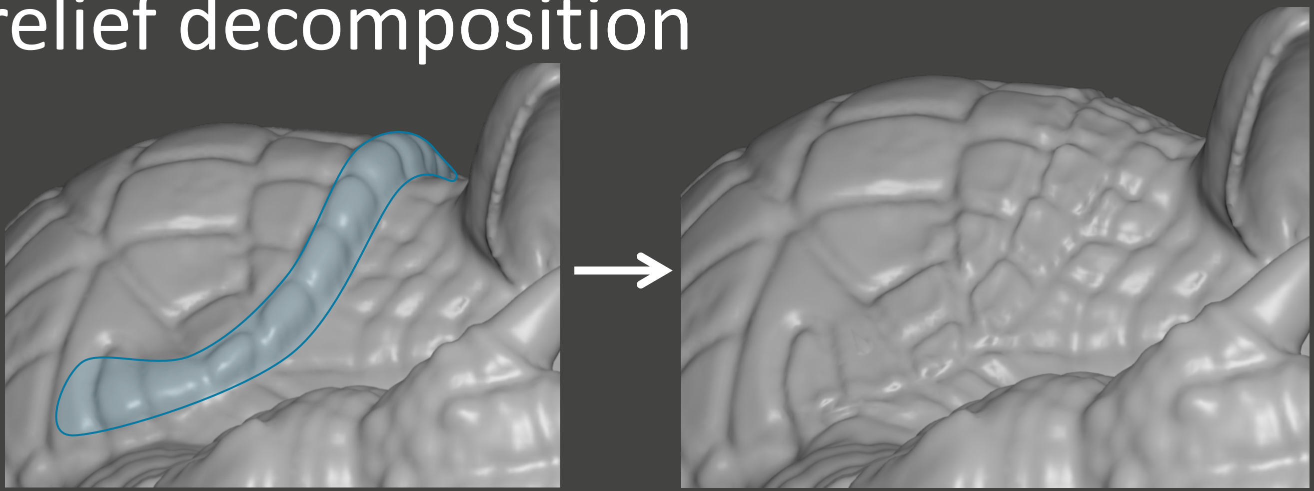




[CGF'12]

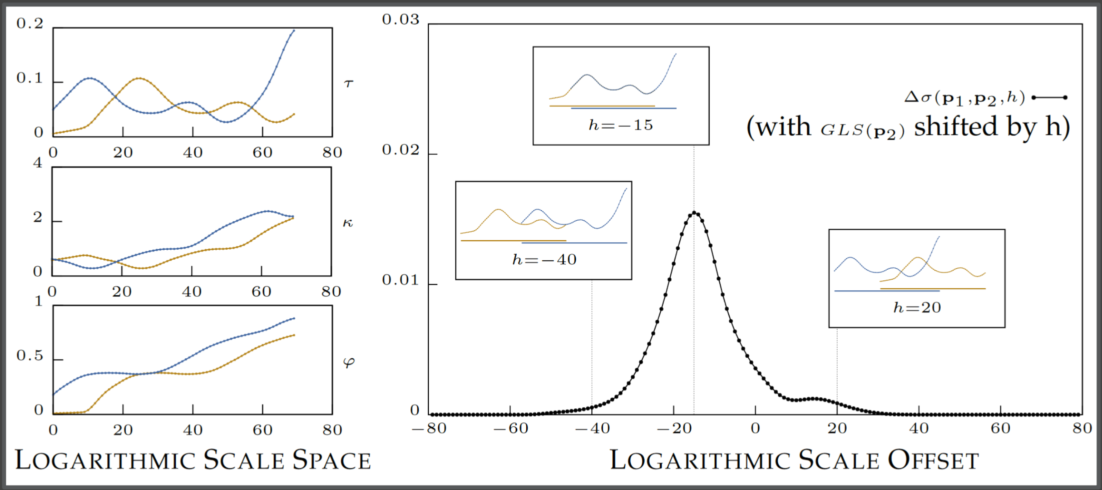
Local analysis: pertinent scale extraction

- Base/relief decomposition



[CGF'14]

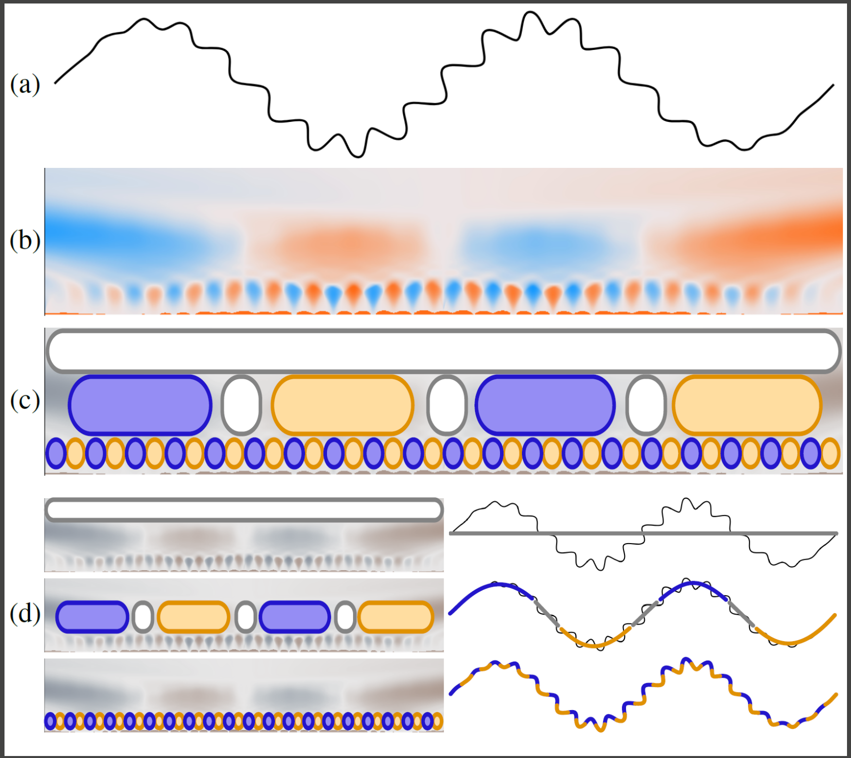
- Relative scale factor estimation



[TVCG'15]

Global analysis: planar primitive extraction

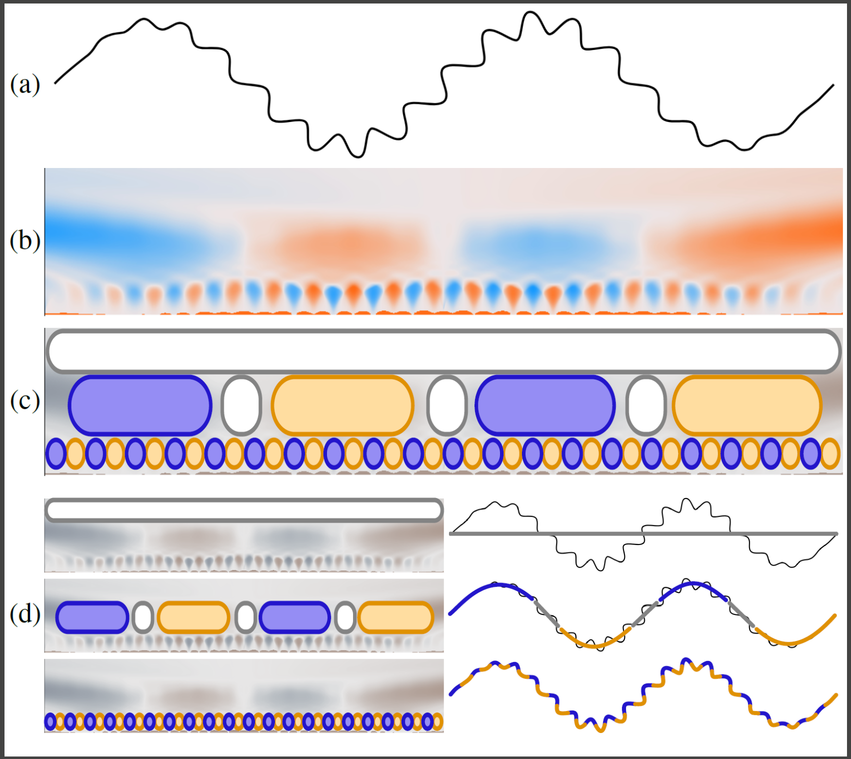
- Spatially regularize the analysis



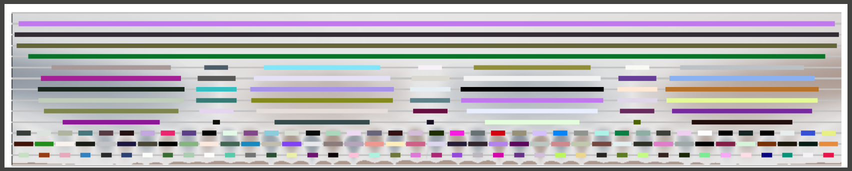
[Eurographics'20]

Global analysis: planar primitive extraction

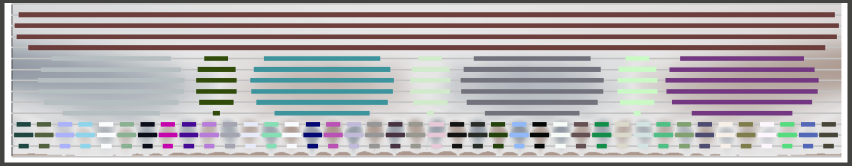
- Spatially regularize the analysis



Stability in space: segmentation



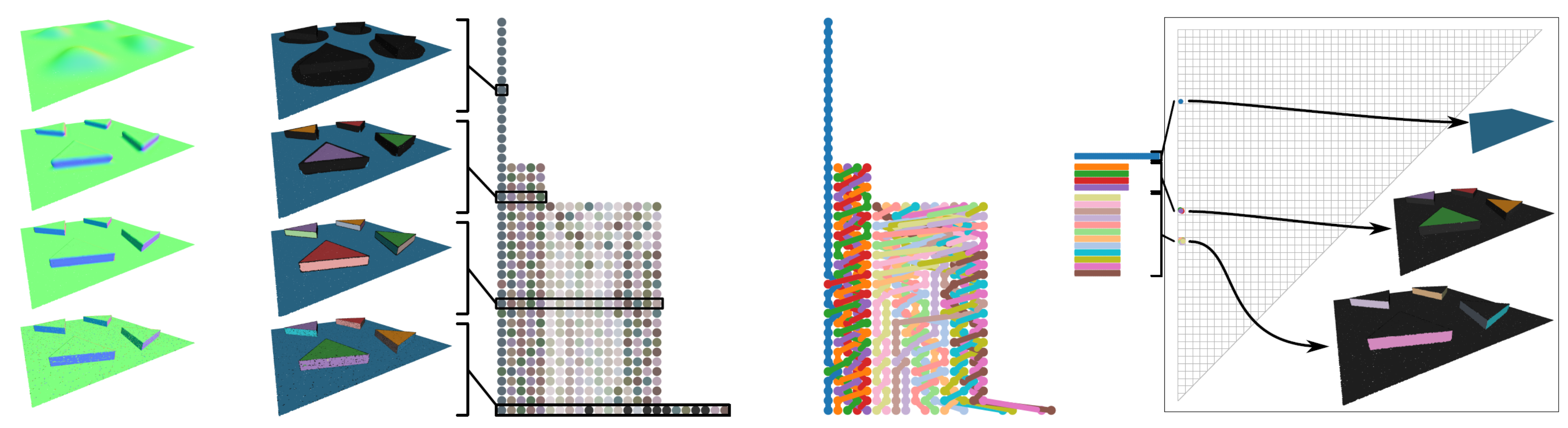
Stability in scale: topological persistence



[Eurographics'20]

Global analysis: planar primitive extraction

- Features defined using topological persistence

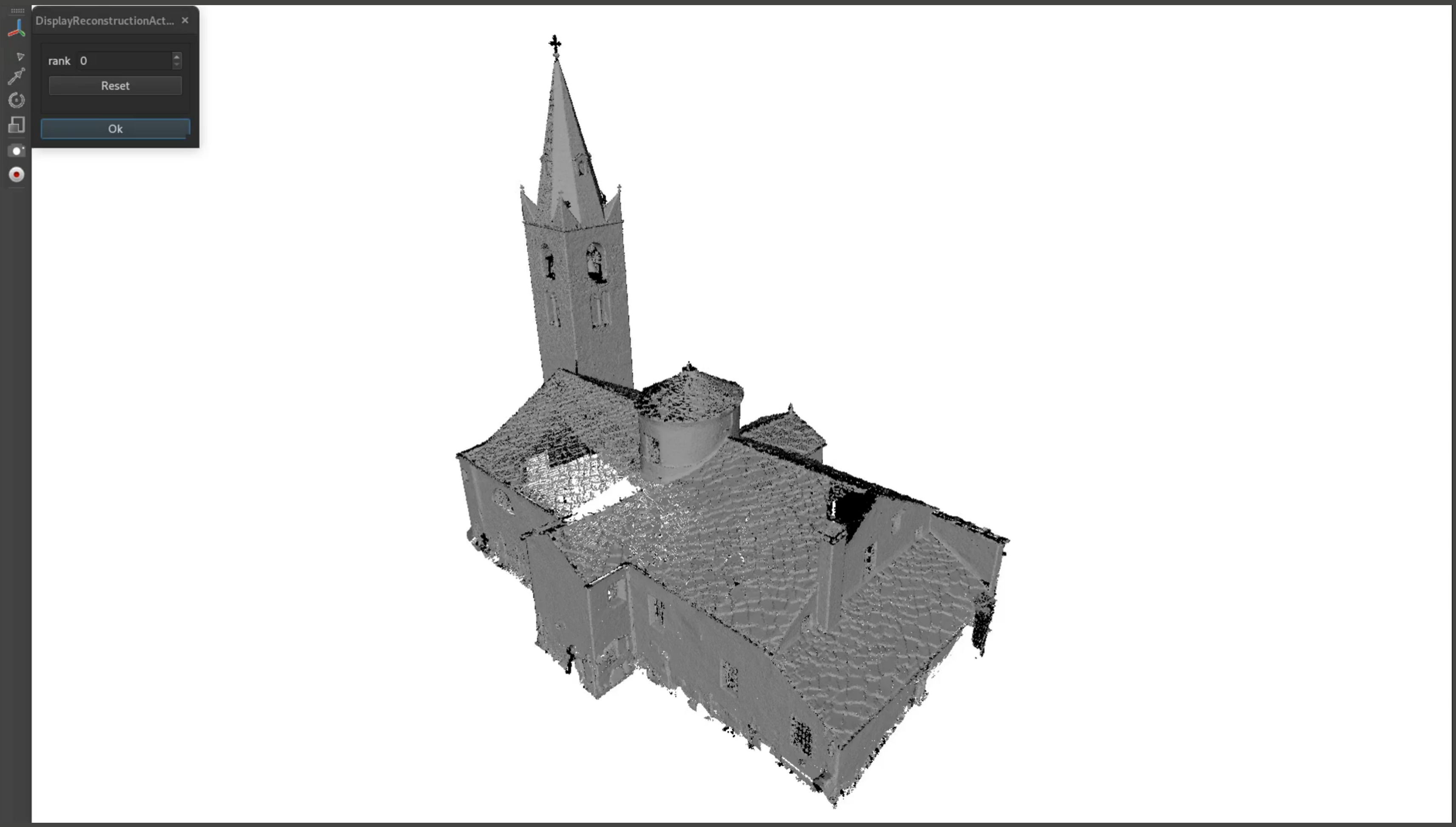


(a) Smoothed input (b) Graph nodes (c) Component extraction (d) Persistence diagram

[Eurographics'20]

Global analysis: planar primitive extraction

- Various shape exploration tools



[Eurographics'20]

Deep learning: classification in scale-space

- Problem: unstructured point clouds do not fit standard network architectures (e.g., CNN)
  - Sampling variation
  - Lack of parameterization
  - Permutations of points

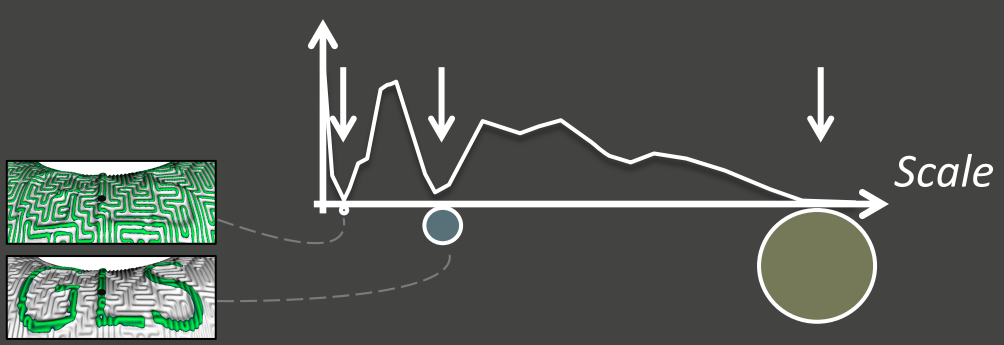
- SOTA:
  - Project on standard convolution kernel
  - Analyse knn graph / patches

Under revision [ToG'21]

Deep learning: classification in scale-space

- Problem: unstructured point clouds do not fit standard network architectures (e.g., CNN)
  - Sampling variation
  - Lack of parameterization
  - Permutations of points

- Idea: use scale-space slice as raw feature vector



Deep learning: classification in scale-space

cnrs

Neural networks for multi-scale edge classification in 3D point cloud

Chems Eddine Himeur [1]

1 : Institute de recherche en informatique de Toulouse (IRIT)
Université Toulouse III - Paul Sabatier

Point clouds are unstructured clusters of data representing geometries. Point cloud analysis is a challenging field, especially challenging for use of neural networks. In this presentation I will explore point cloud analysis using neural networks, and the combination between scale-space analysis and neural network classifier, to make a robust and fast edge detection network for point cloud labeling.

- Teaser:
  - Learn and classify at interactive rates
  - Requires very small data and computational power
  - Produce better classification than state of the art

Under revision [ToG'21]

Take home message

- Implicit Scale-Space
  - bring ideas from scale-space analysis,
  - to unstructured point-based data,
  - using implicit surface representation

- Expectations
  - Define reliable, fast and robust analysis and processing tools
  - For complex shapes
  - Represented as massive point-clouds

- What's next ?
  - Improve estimators: convergence guaranties, evaluation speed
  - Improve evaluation scheme on real data https://3dard.cnrs.fr

REFERENCES

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- *Adaptive multi-scale analysis for point-based surface editing*
  Georges Nader; Gael Guennebaud; Nicolas Mellado
  Pacific Graphics (2014).
- *Relative scale estimation and 3D registration of multi-modal geometry using Growing Least Squares*
  Nicolas Mellado; Matteo Dellepiane; Roberto Scopigno
  Transactions on Visualization and Computer Graphics (2016).
- *Persistence Analysis of Multi-scale Planar Structure Graph in Point Clouds*
  Thibault Lejemble; Claudio Mura; Loic Barthe; Nicolas Mellado
  Eurographics 2020
- *PCEDNet : A Light-Weight Neural Network for Fast and Interactive Edge Detection in 3D Point Clouds*
  Under review at ACM Transactions on Graphics