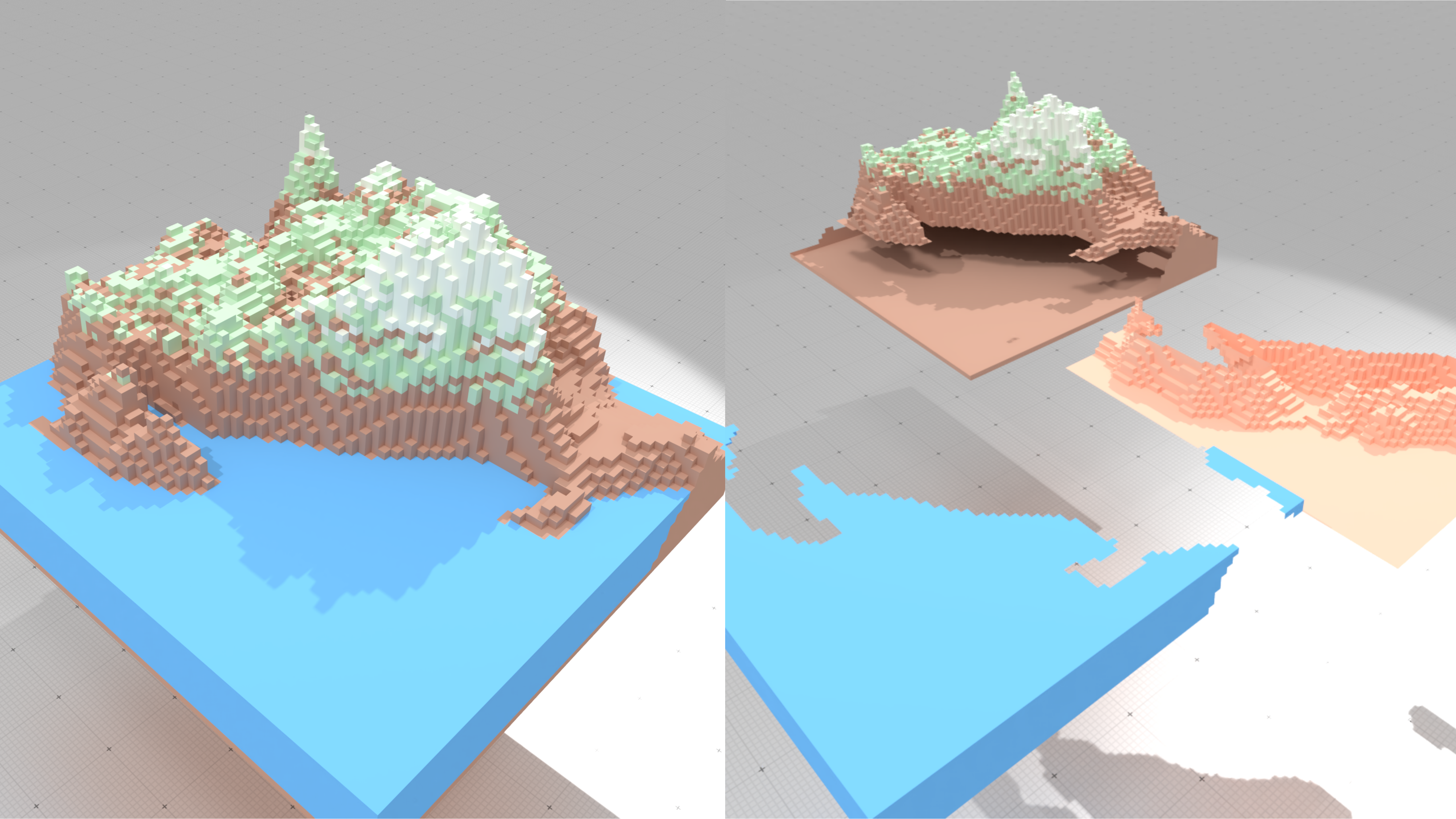
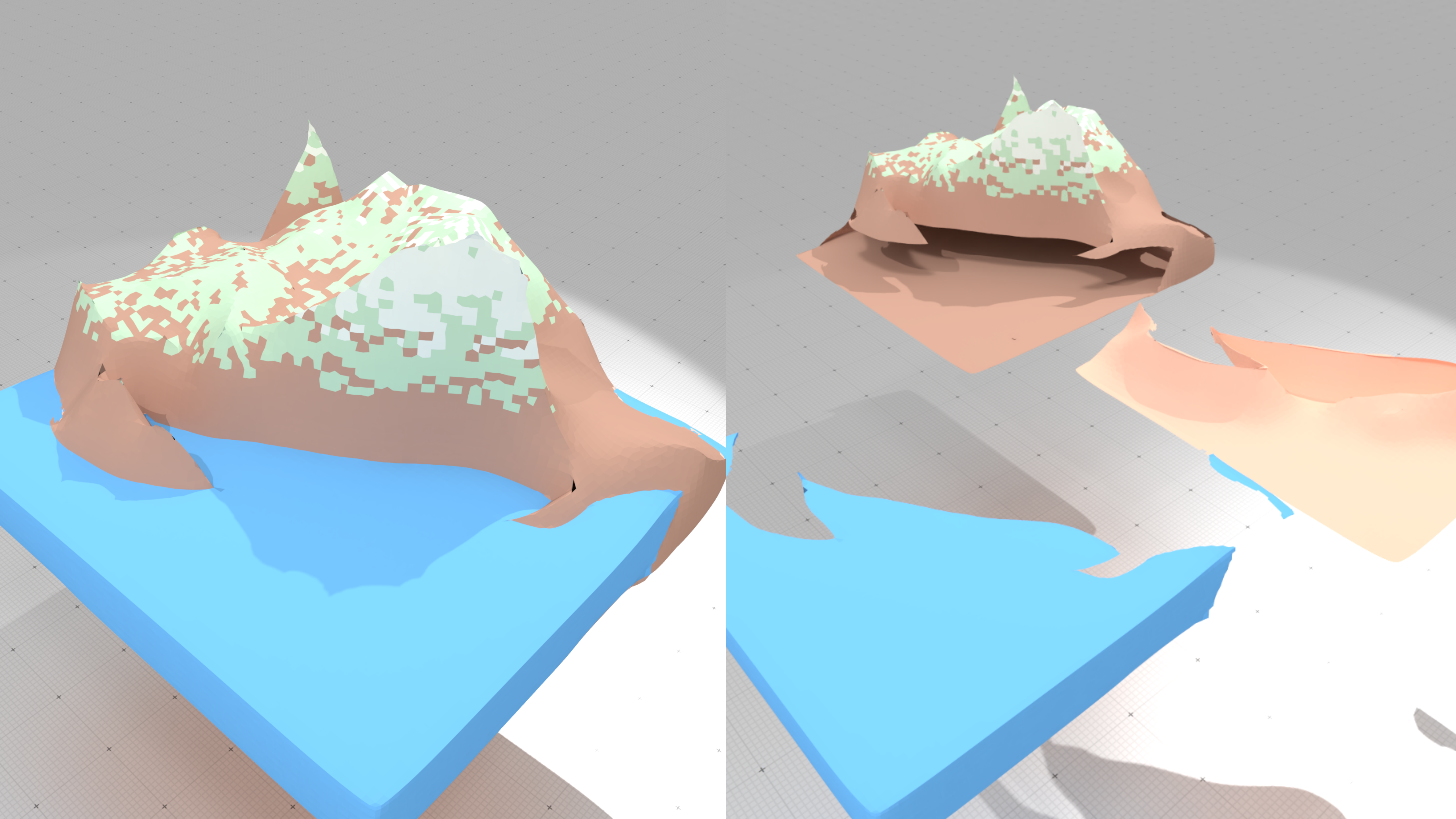
DIGITAL SURFACE REGULARIZATION WITH GUARANTEES

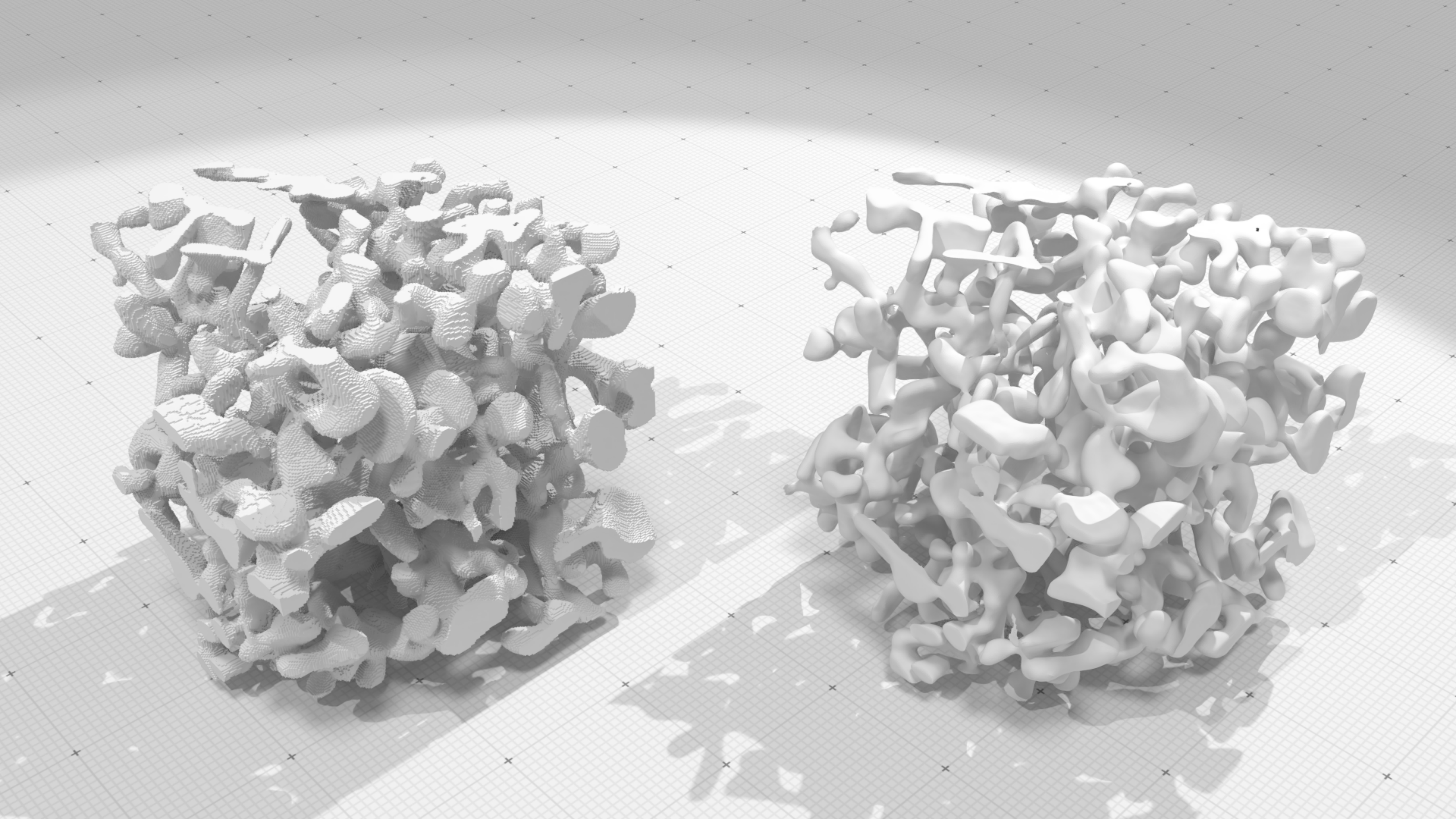
David Coeurjolly, *CNRS, Lyon*

Pierre Gueth, *Adobe*

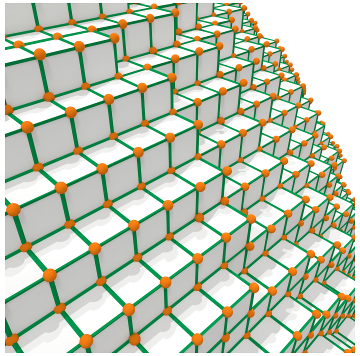
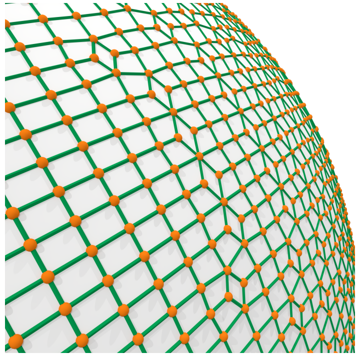
Jacques-Olivier Lachaud, *Univ. Savoie Mont-Blanc*







OBJECTIVES

Regularize the surface of a voxel set

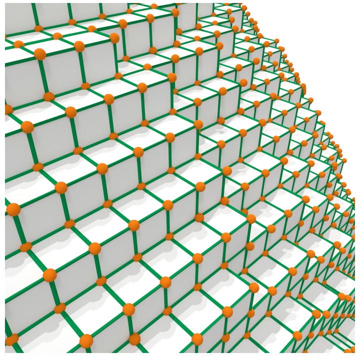
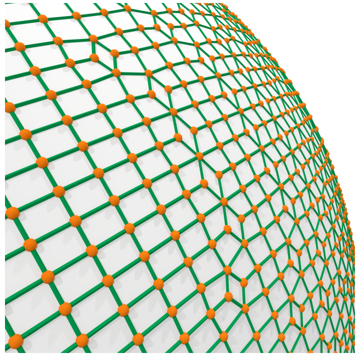
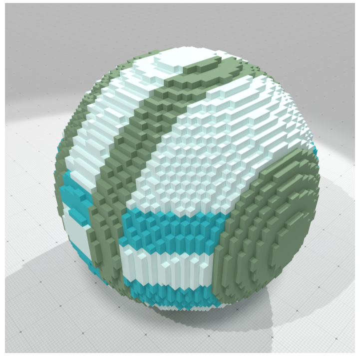
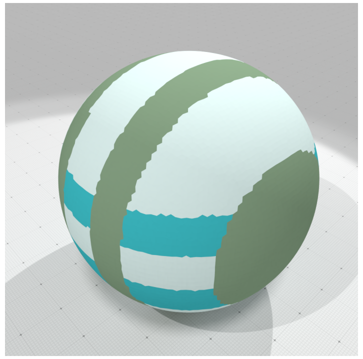
with the same combinatorics,

OBJECTIVES

Regularize the surface of a voxel set

with the same combinatorics,

with a voxel attributes mapping,

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Regularize the surface of a voxel set

with the same combinatorics,

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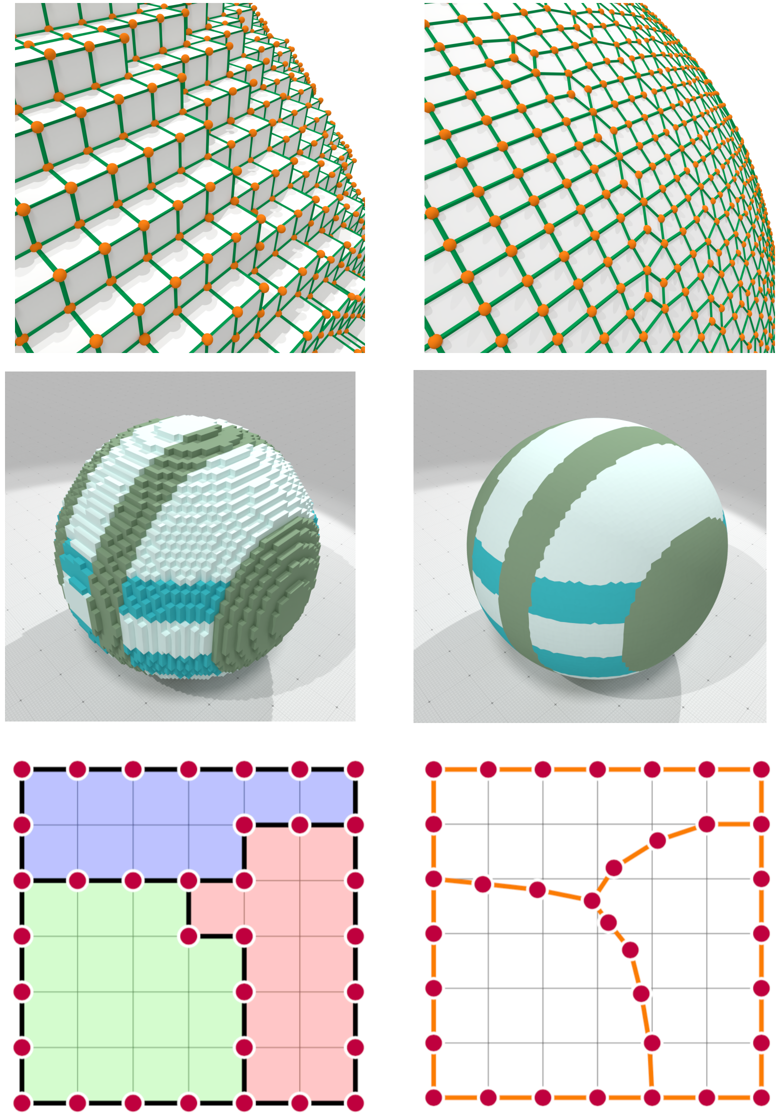
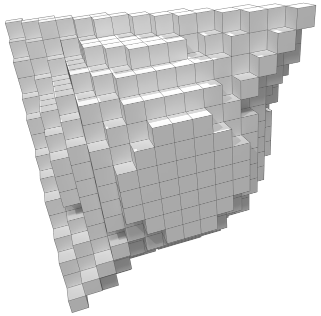
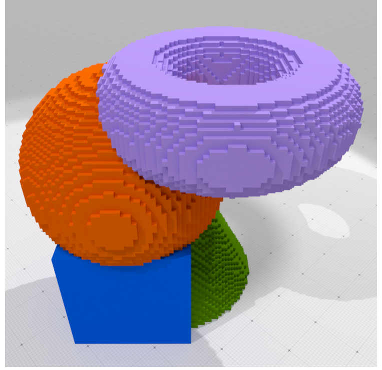
on labeled image interfaces



RELATED WORKS





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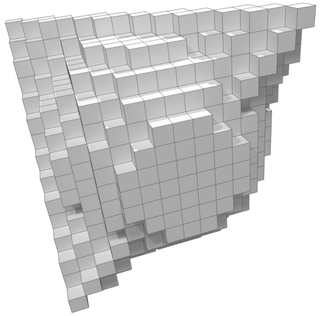
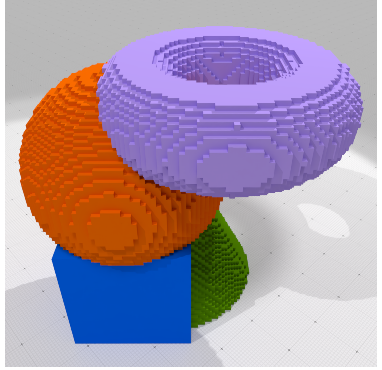


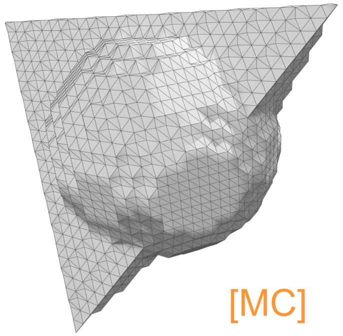


Iso-contouring approaches [Marching-Cubes (MC), Dual-contouring (DC)…]

local construction of the mesh with fast algorithms (GPU friendly, multi-labeled images, adaptive…). Great for implicit functions / SDF

😕 sensitive to noise

😕 [DC] requires high quality Hermite data (position and normal vector)



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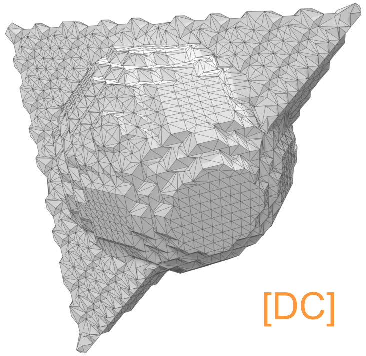

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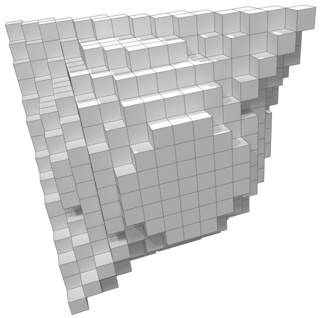

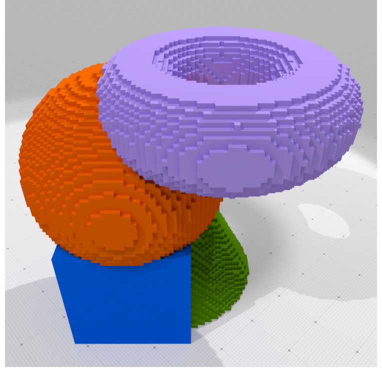

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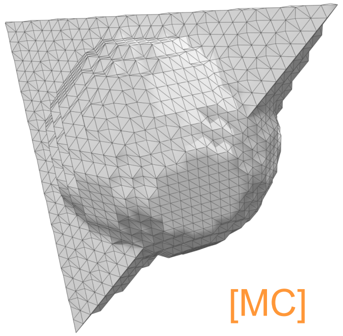

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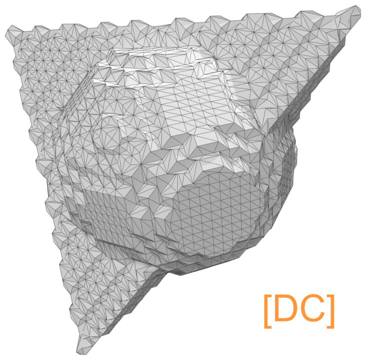

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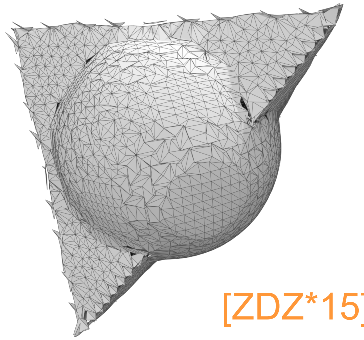

[ZDZ*15]

Surface denoising [HS13, WYL* 14, WZCF15, ZWZD15, ZDZ*15]

extract an iso-surface and apply feature preserving denoising

😕 remeshing may lose the mapping with the original voxel data

😕 sensitive to noise or low resolution voxel shapes

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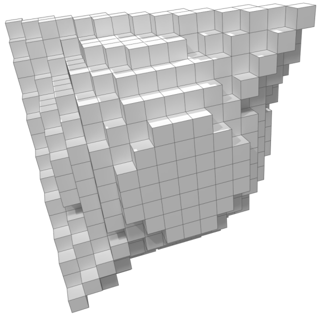

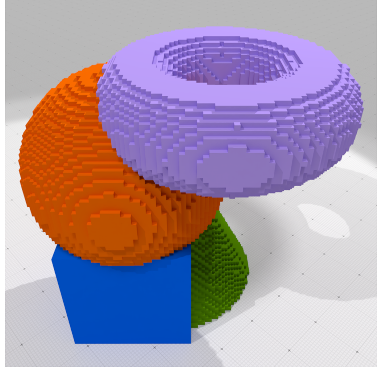

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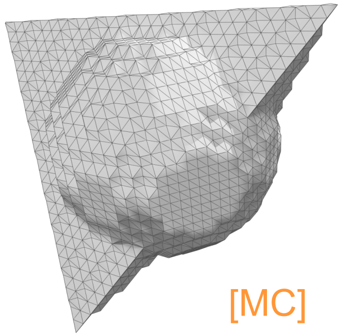

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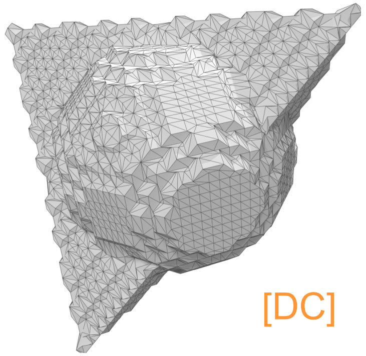

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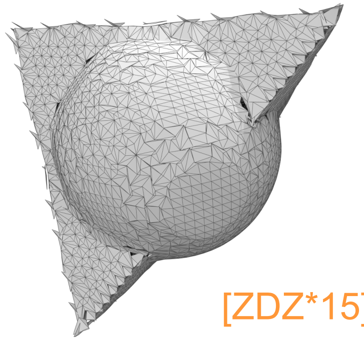
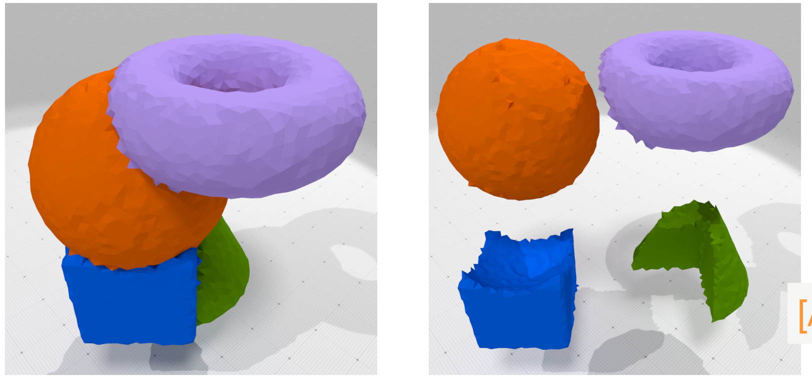


[ZDZ*15]

Volumetric reconstruction [LS07, DVS* 09, BYB09, BLW13, FTB16, AJR*17]

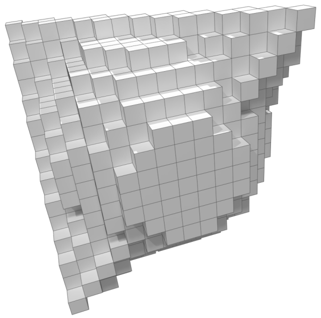
variational formulation to optimize the geometry of tetrahedra while preserving interfaces

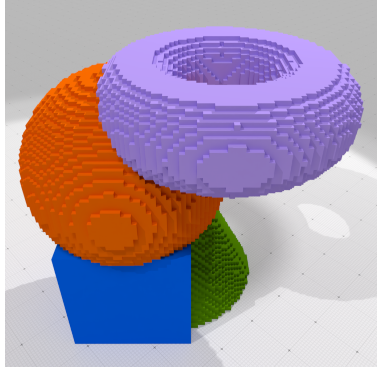
😕 non smooth interfaces for low resolution voxel shapes



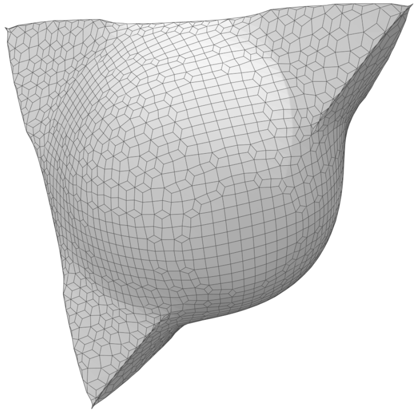
[AJR*17]

CONTRIBUTIONS

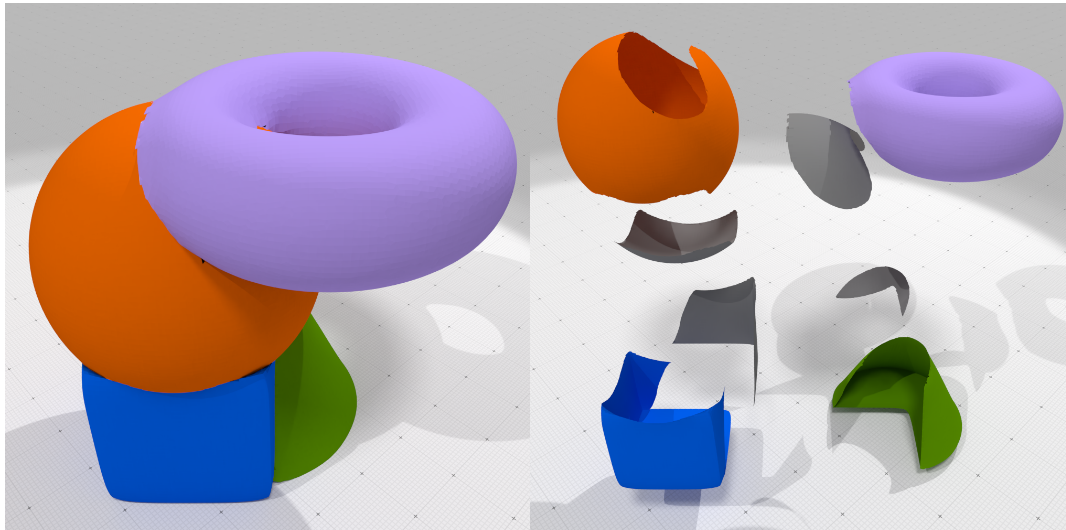


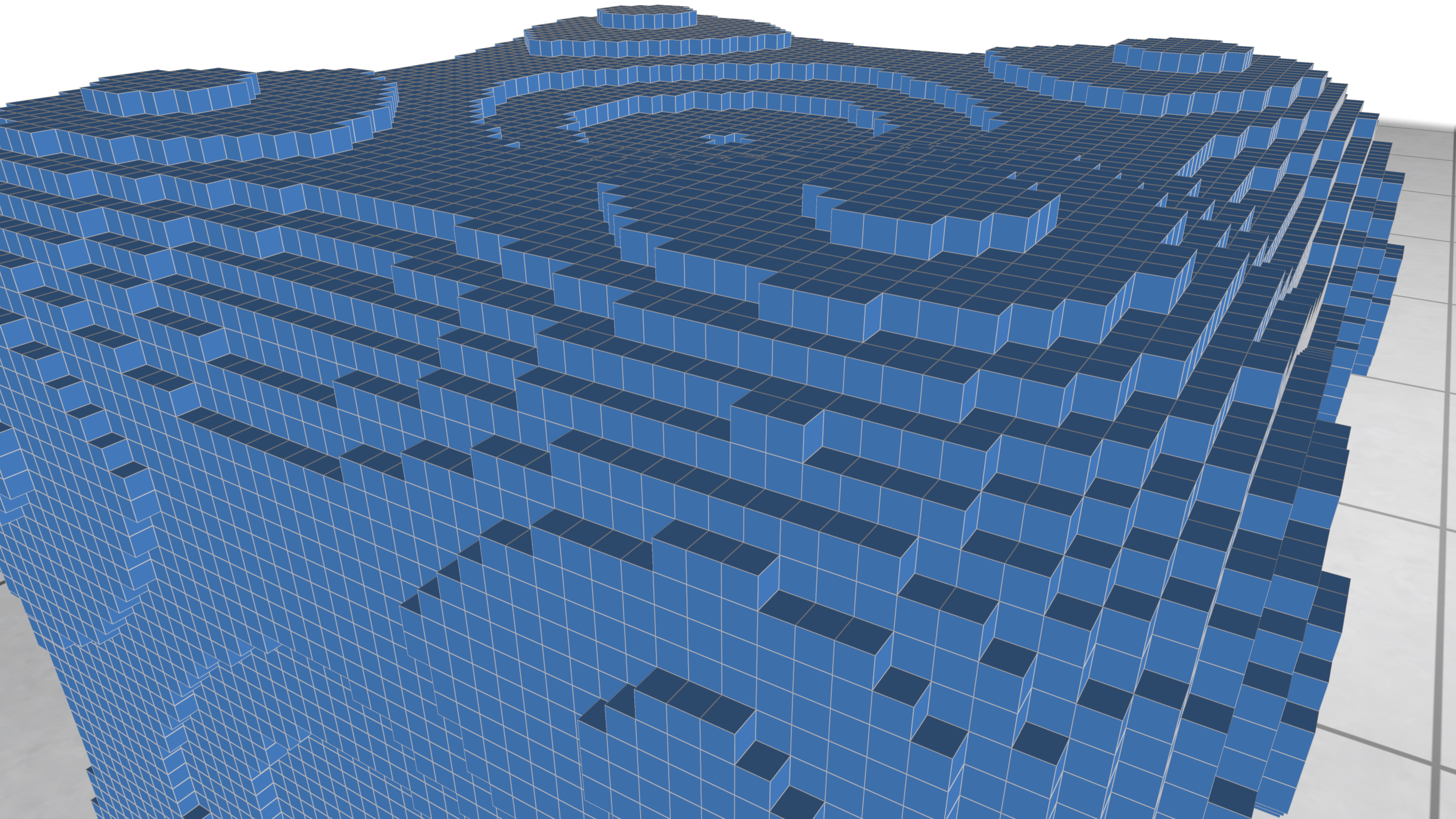


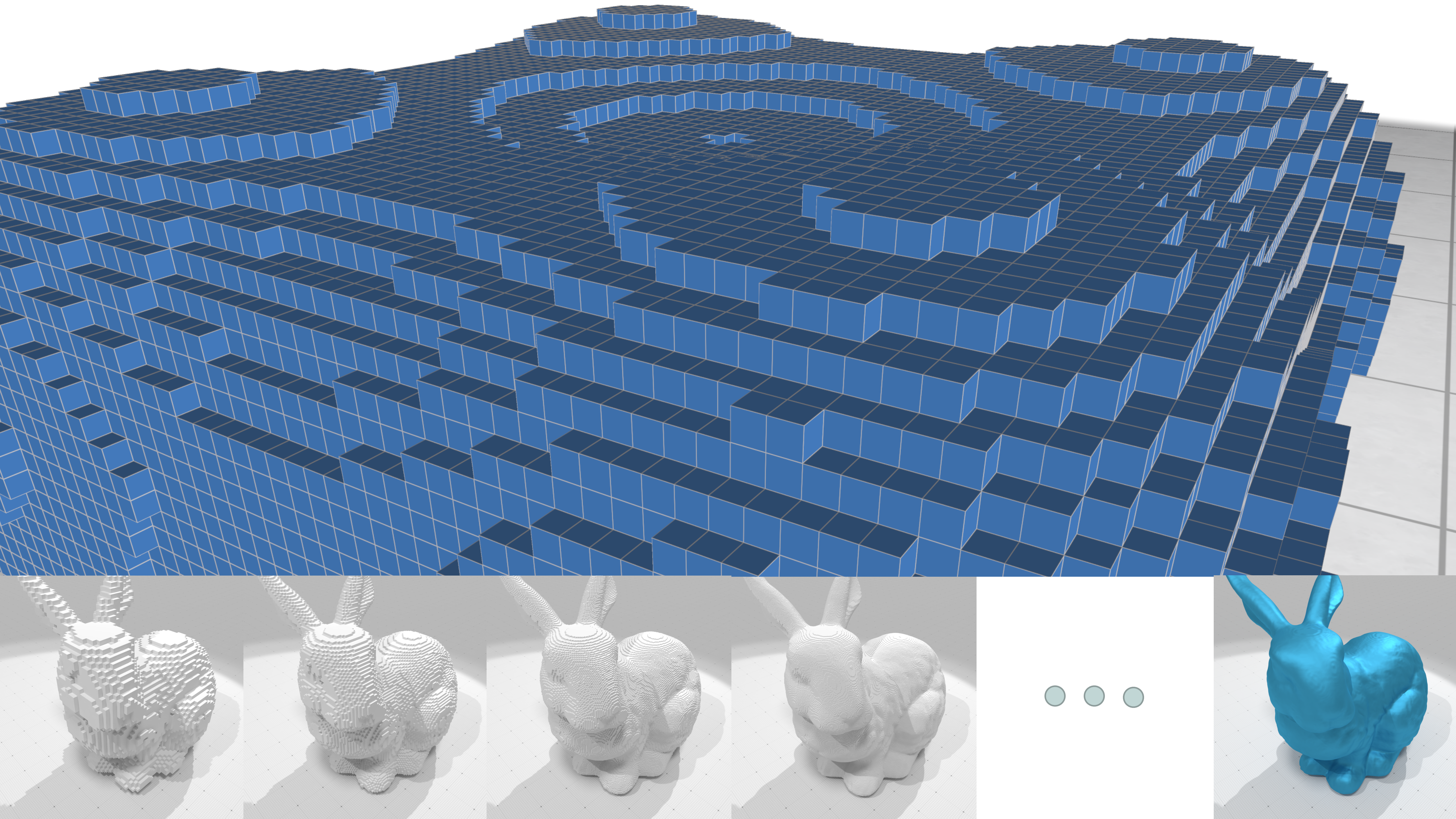
Digital surface regularization:

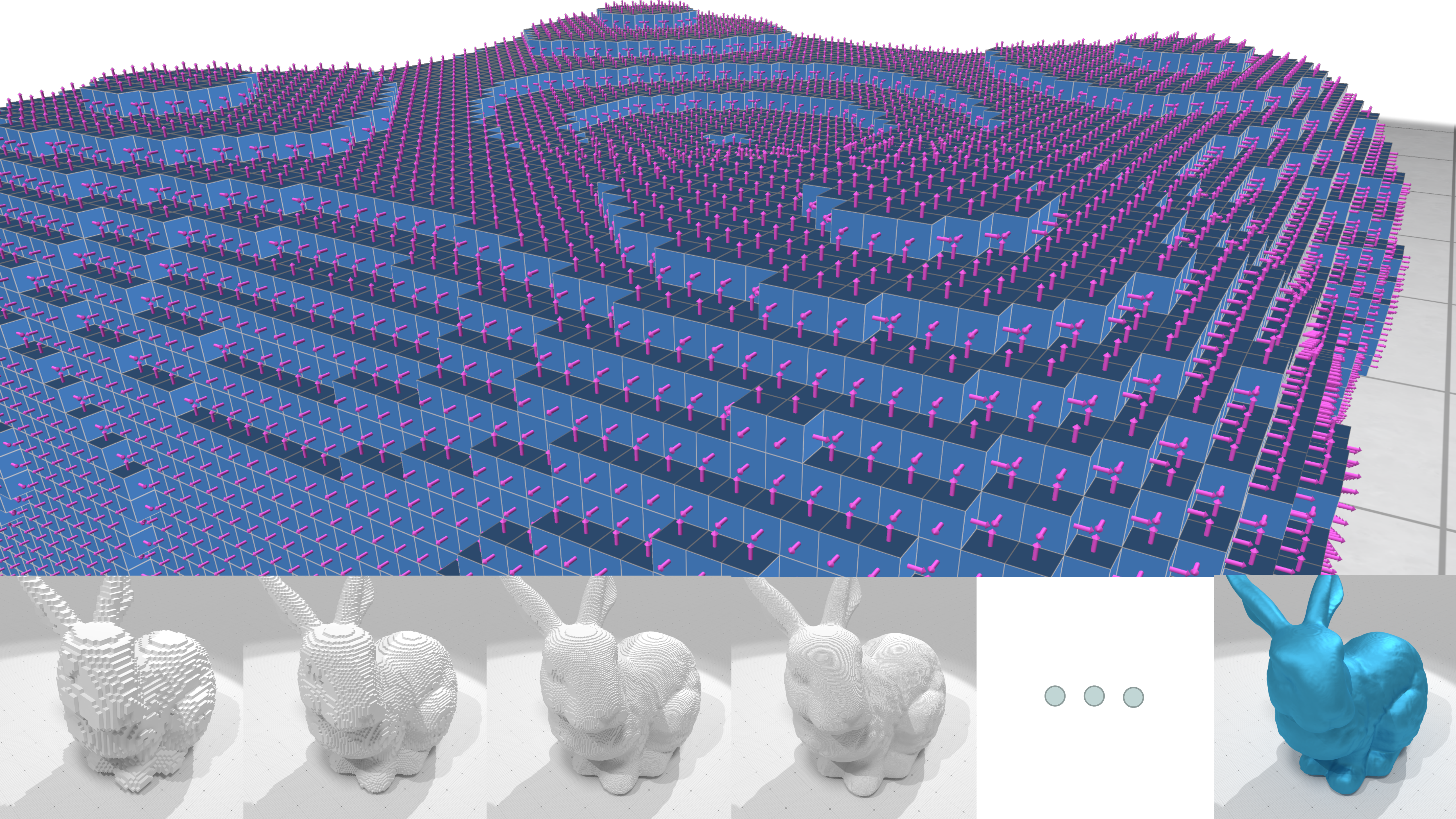


- robust from low to high-res, w/o noise
- easy to implement (convex energy function, GPU solvers)
- one-to-one mapping with input quads
- multi-labeled images
- stability results thanks to multigrid convergence

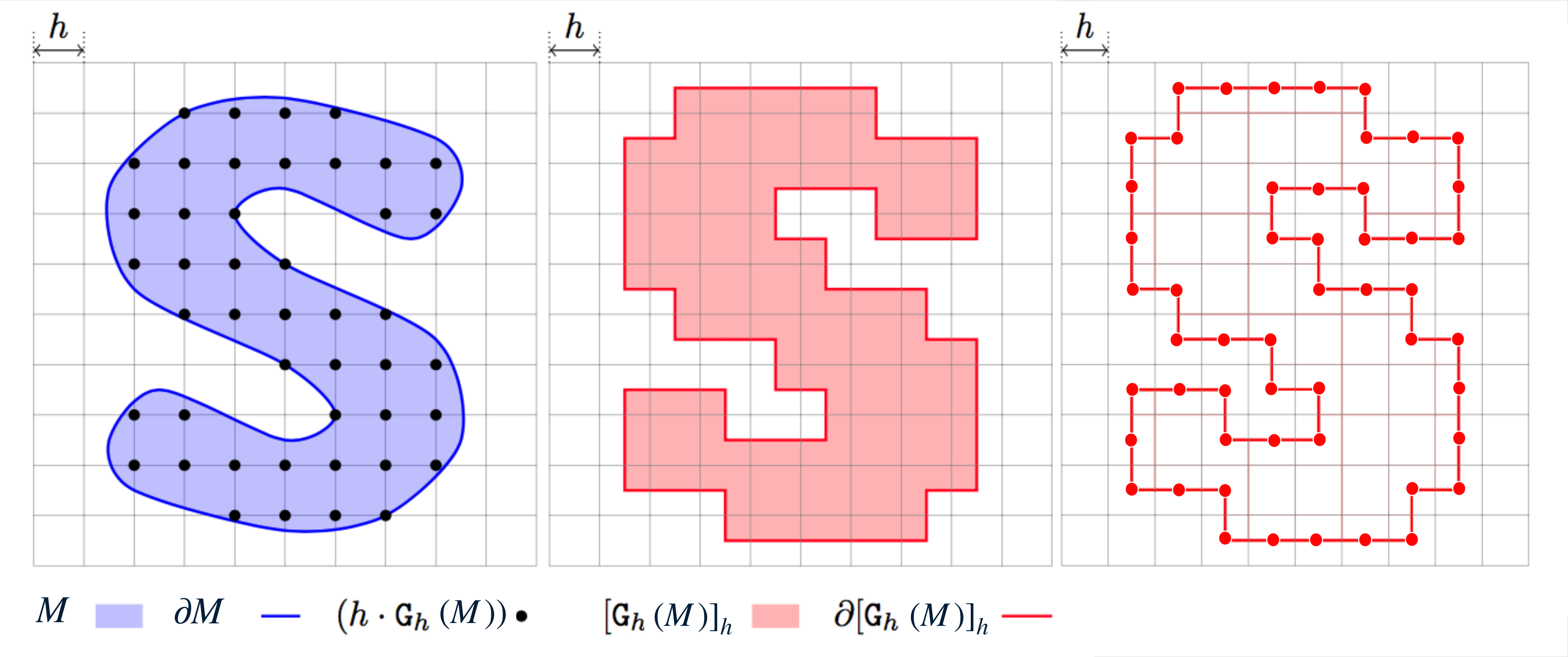








GEOMETRY PROCESSING ON DIGITAL DATA



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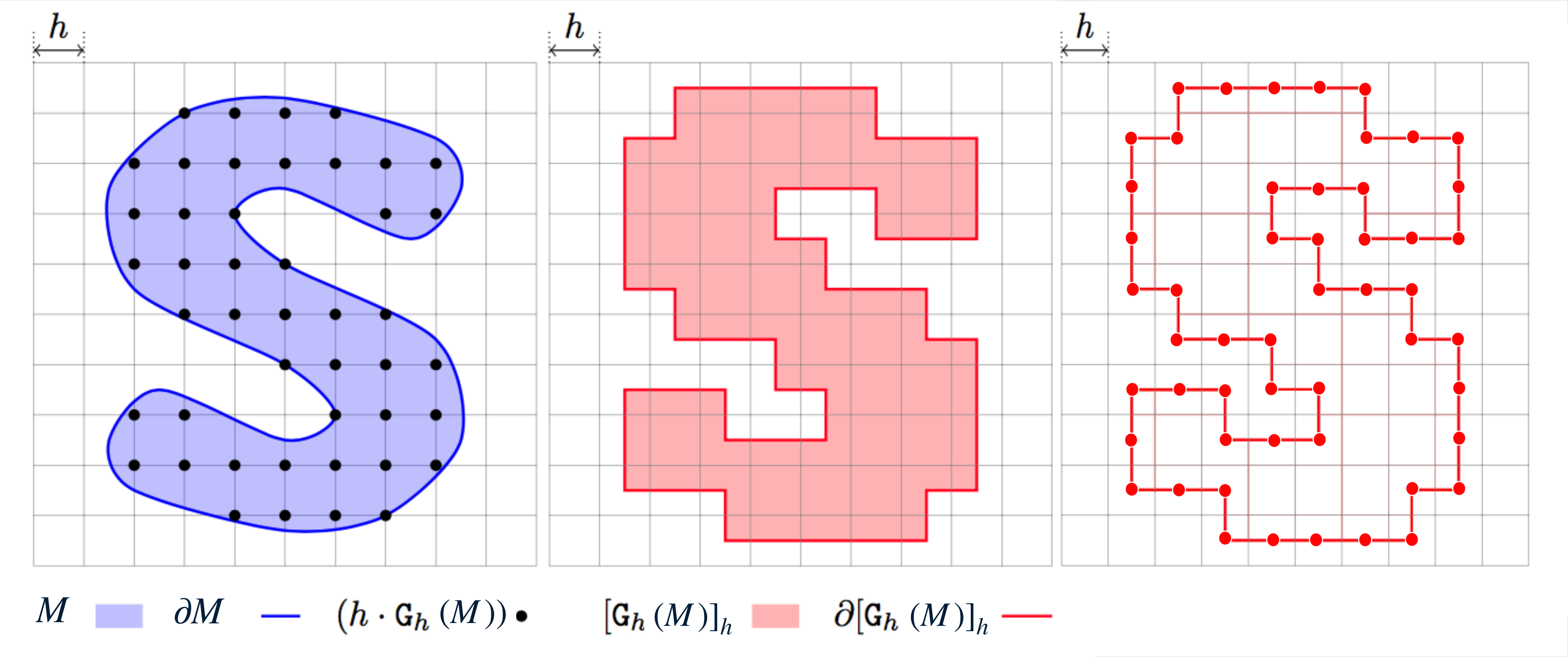
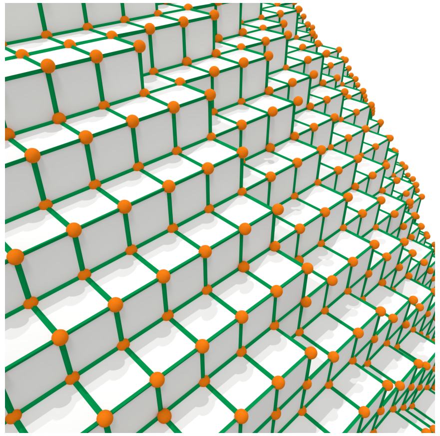
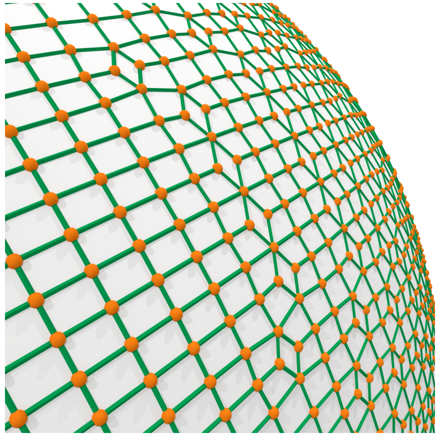


For any compact domain $M \in \mathbb{R}^d$ such that ∂M has positive reach, and its digitization M_h on a grid with grid-step h, then $d_H(\partial M, \partial M_h) \leq \sqrt{d}/2h$ and the canonical projection map is one-to-one almost everywhere as h tends to zero.

[LT16]

VARIATIONAL FORMULATION





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Data attachment term: points stay close to the original surface

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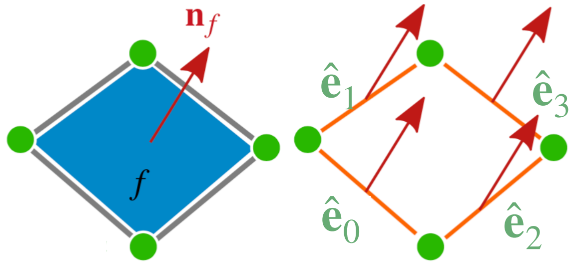
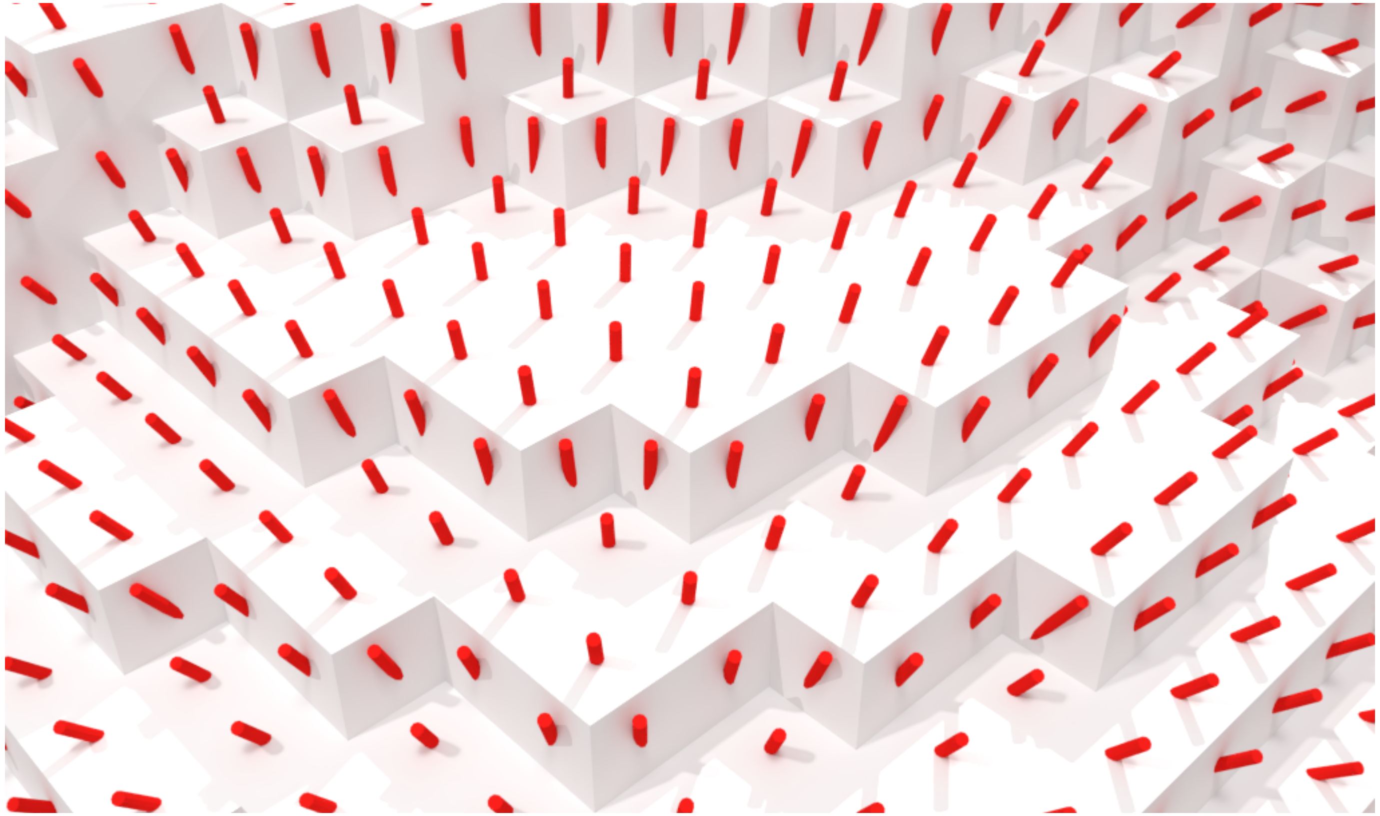


NORMAL VECTOR FIELD ESTIMATION

Normal vector per quad:

- Multigrid convergent estimation [CLL14]
- w/o feature preserving piecewise smooth reconstructions [BM12, CFGL16]



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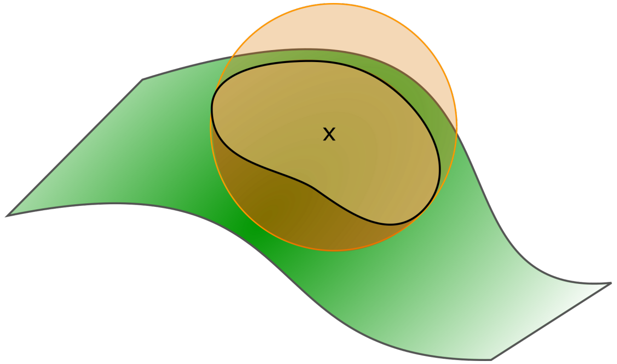
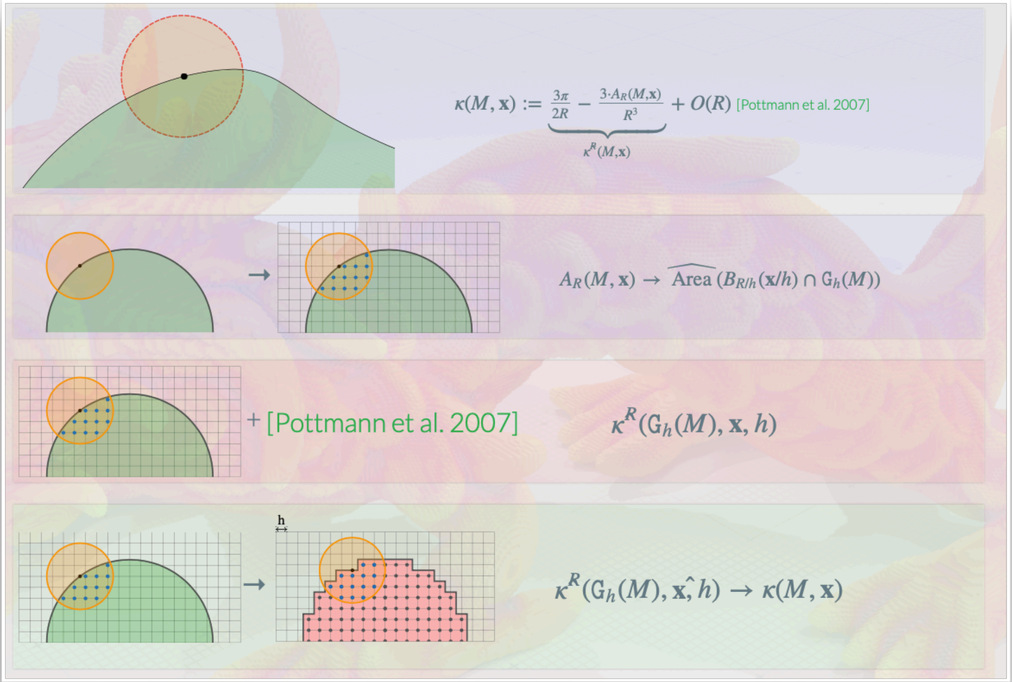


[CLL14] [LCL17] [LRTC20]



[C., Levallois, Lachaud]

Let M be a convex shape in $\mathbb{R}^2$ with a C^3 bounded positive curvature boundary.

$$\forall \mathbf{x} \in \partial M, \forall \hat{\mathbf{x}} \in \partial[\mathbb{G}_h(M)]_h, \|\hat{x} - x\|_\infty \leq h \Rightarrow$$

$$|\kappa^R(\mathbb{G}_h(M), \hat{\mathbf{x}}, h) - \kappa(M,\mathbf{x})| = O(R) + O\left(\frac{h^\beta}{R^{1+\beta}}\right) + O\left(\frac{h^{\alpha'}}{R^2}\right) + O\left(h^{\alpha'}\right) + O\left(\frac{h^{2\alpha'}}{R^2}\right)$$

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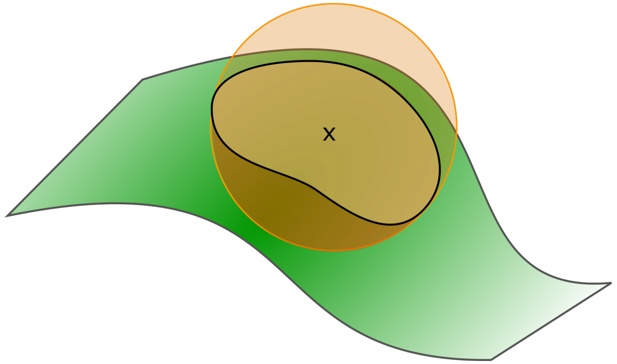

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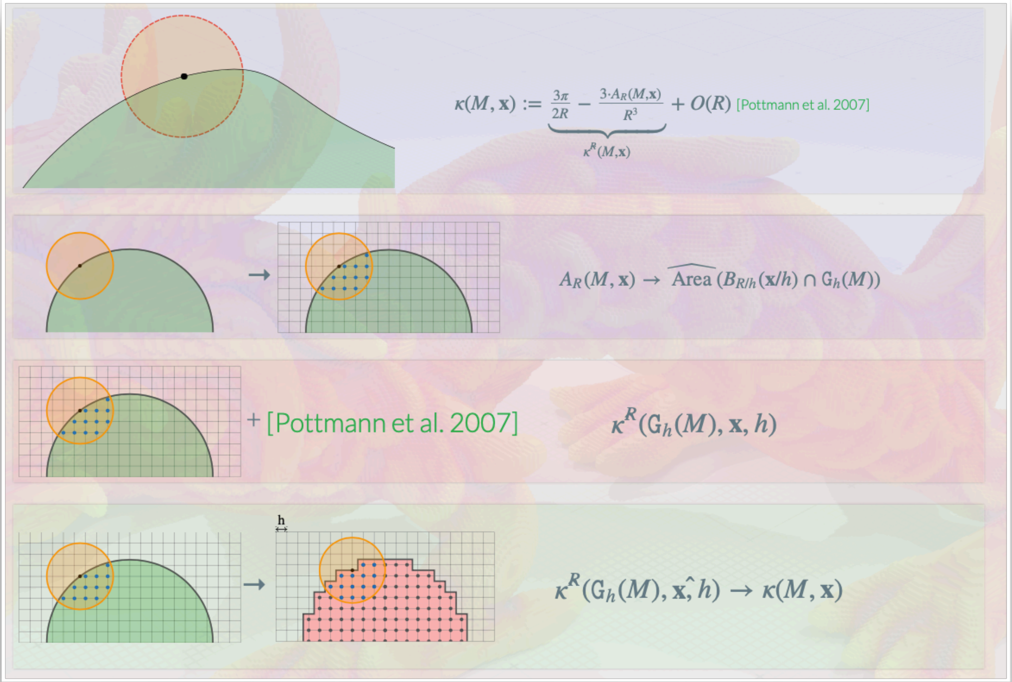

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$$\ldots \left\| \hat{\mathbf{n}}(M_h, \xi(x)) - \mathbf{n}(M,x) \right\|_2 \leq C \cdot h^{\frac{2}{3}}$$

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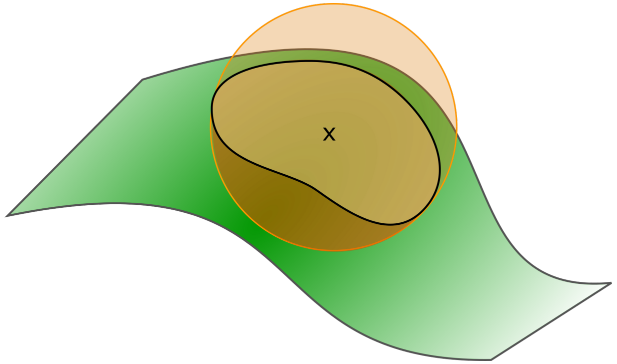

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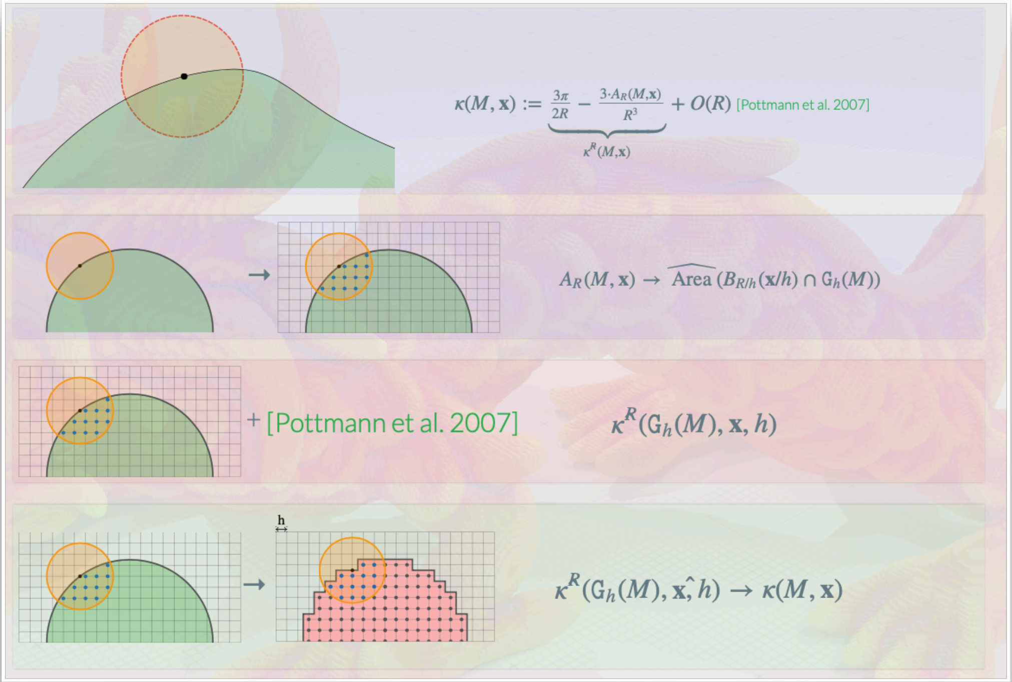

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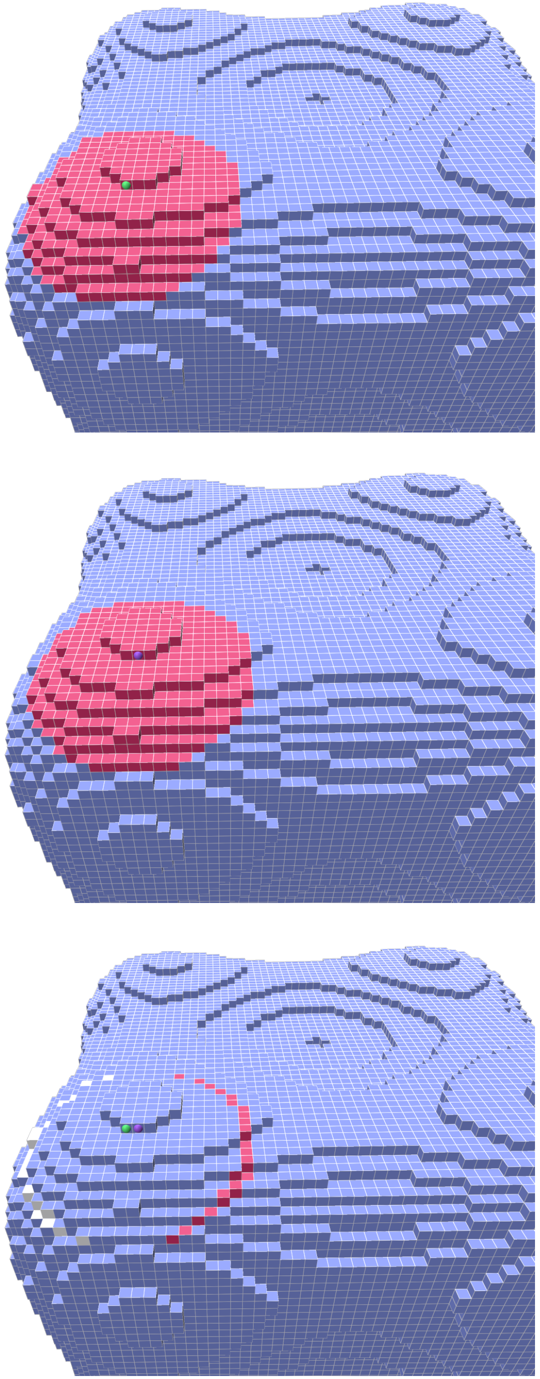

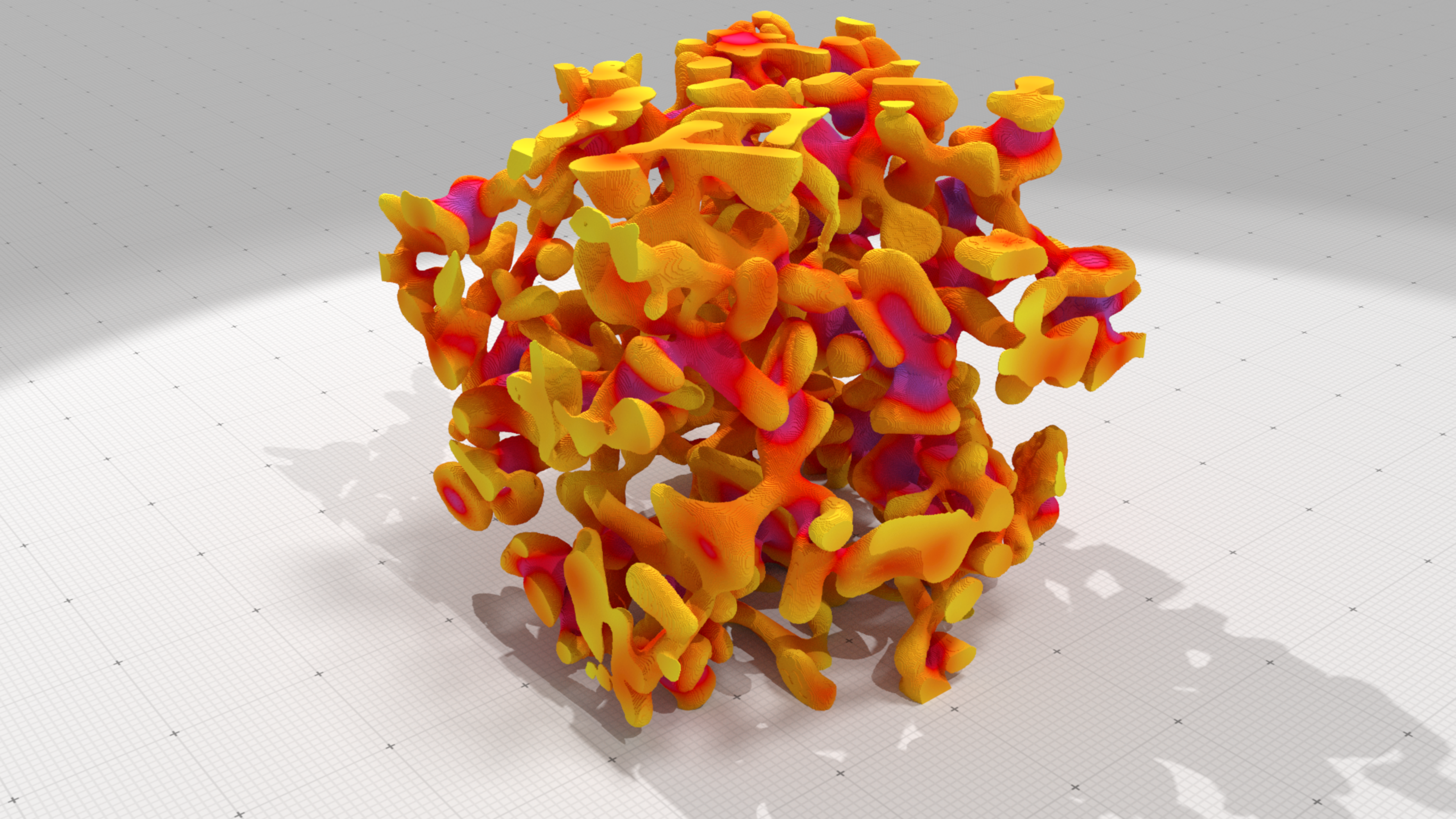

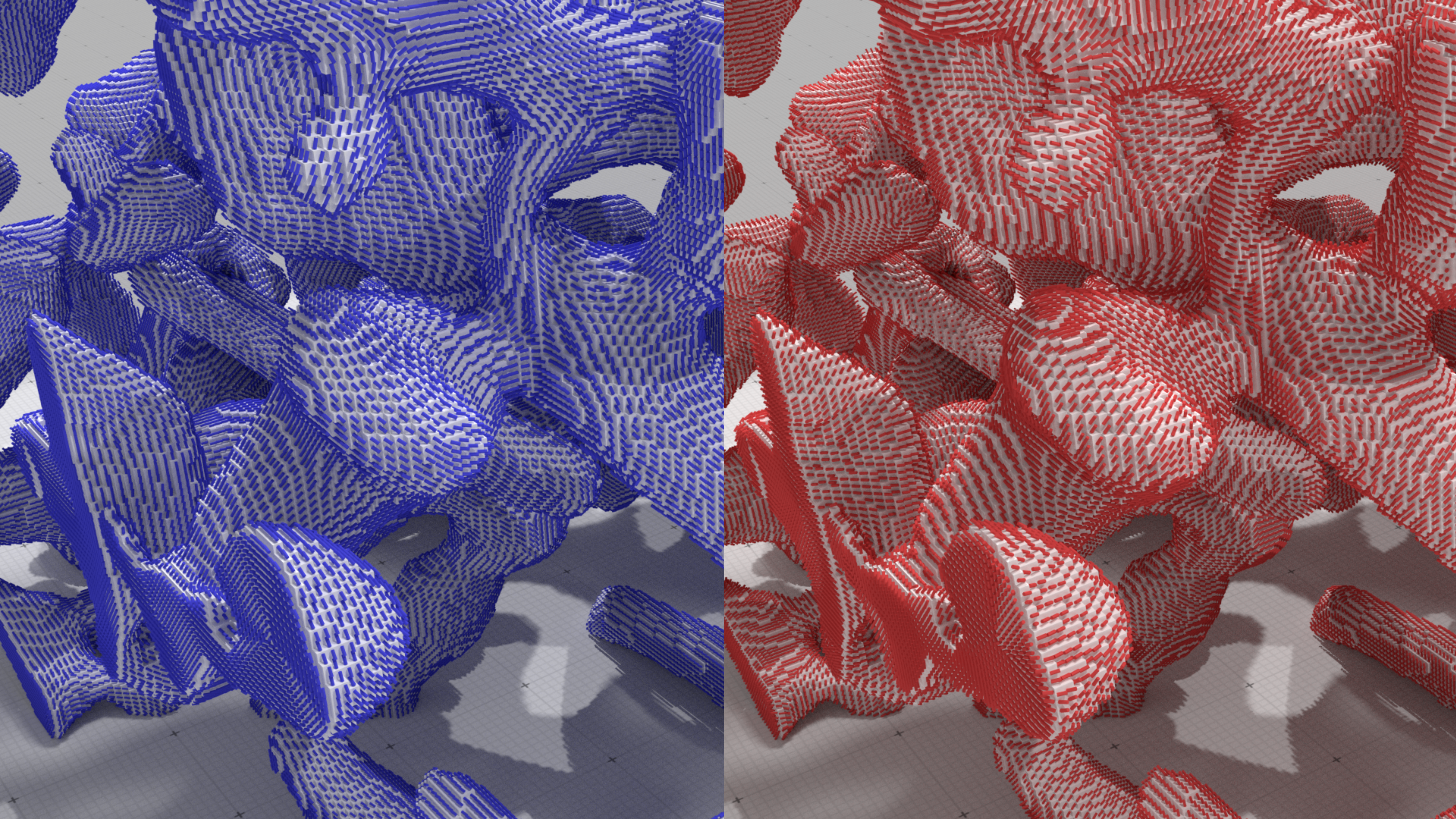

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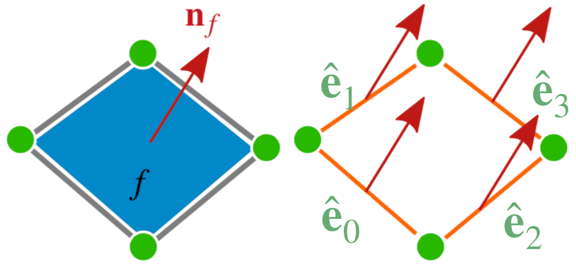

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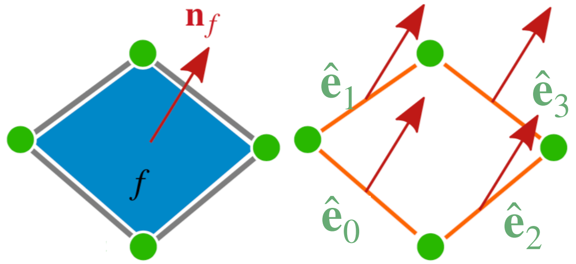
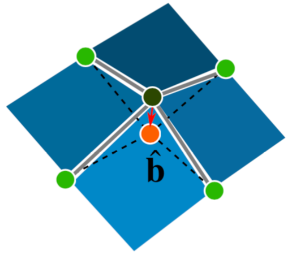
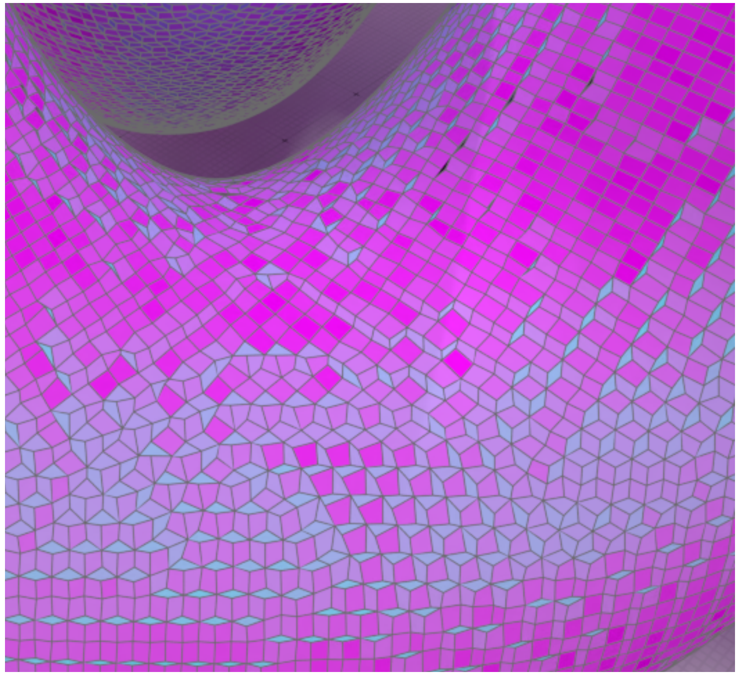
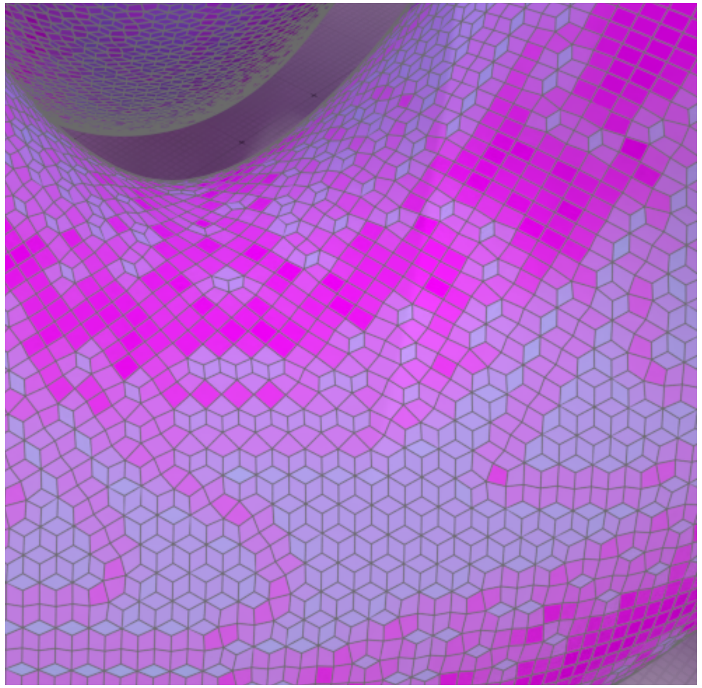
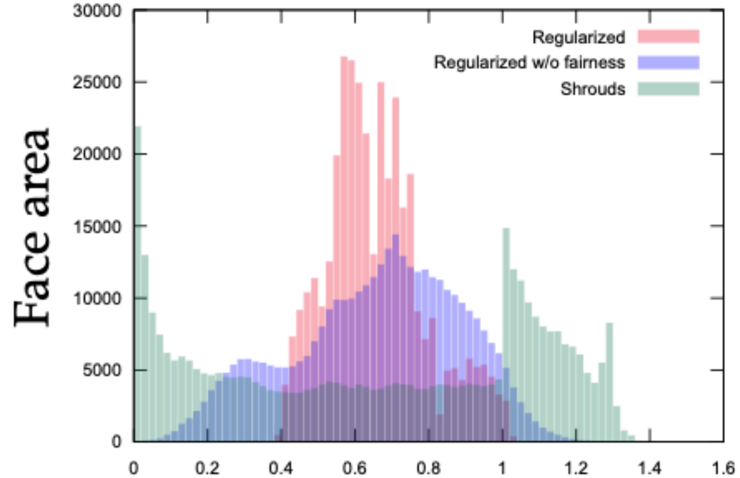


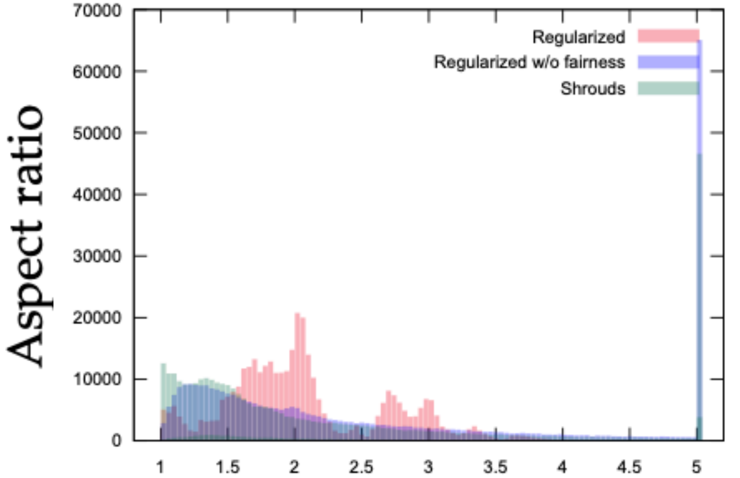
Fairness term: forces the points to be close to their neighbors barycenter








Face area
Regularized
Regularized w/o fairness
Shrouds
30000
25000
20000
15000
10000
5000
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6


Aspect ratio
Regularized
Regularized w/o fairness
Shrouds
70000
60000
50000
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30000
20000
10000
0
1
1.5
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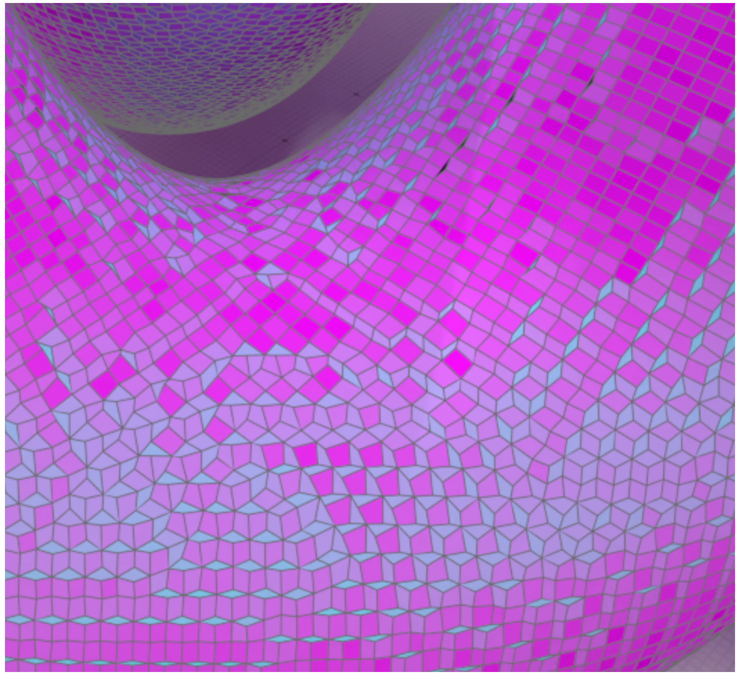
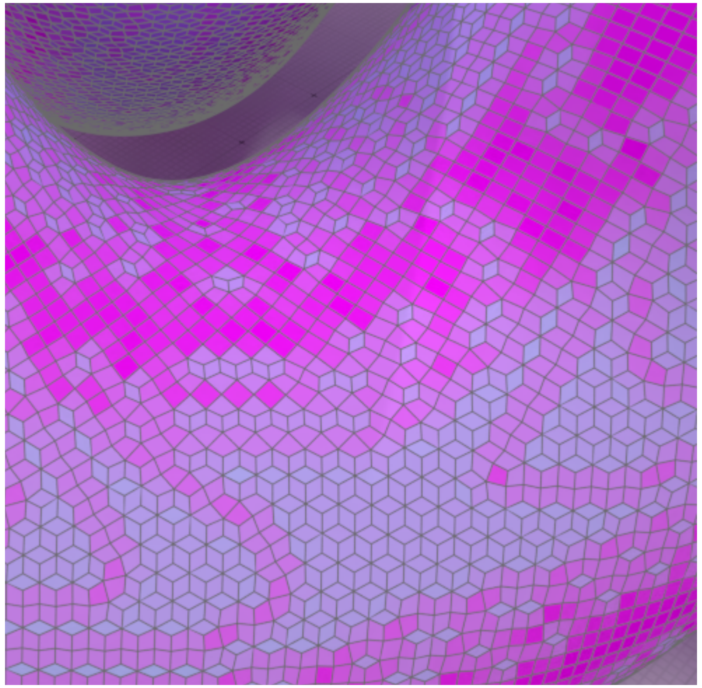
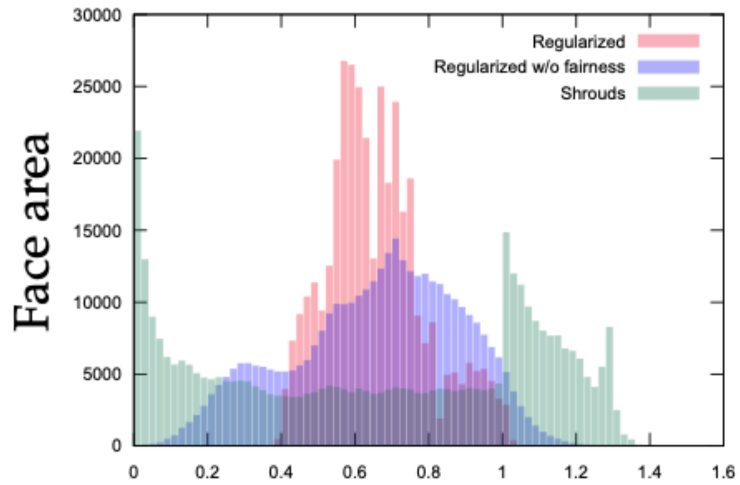
3.5
4
4.5
5

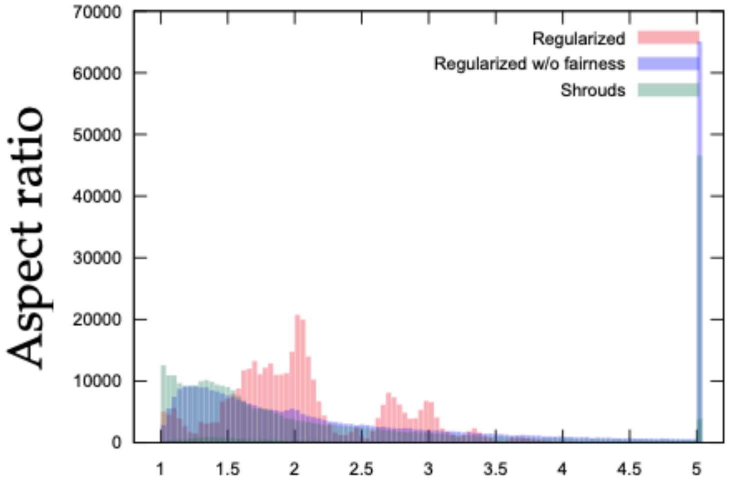


without fairness term



with fairness term





DISCRETIZATION & MINIMIZATION

$$P^* = \underset{\hat{P}}{\operatorname{argmin}}\ \mathscr{E}(\hat{P})$$

Convex energy with explicit gradients:

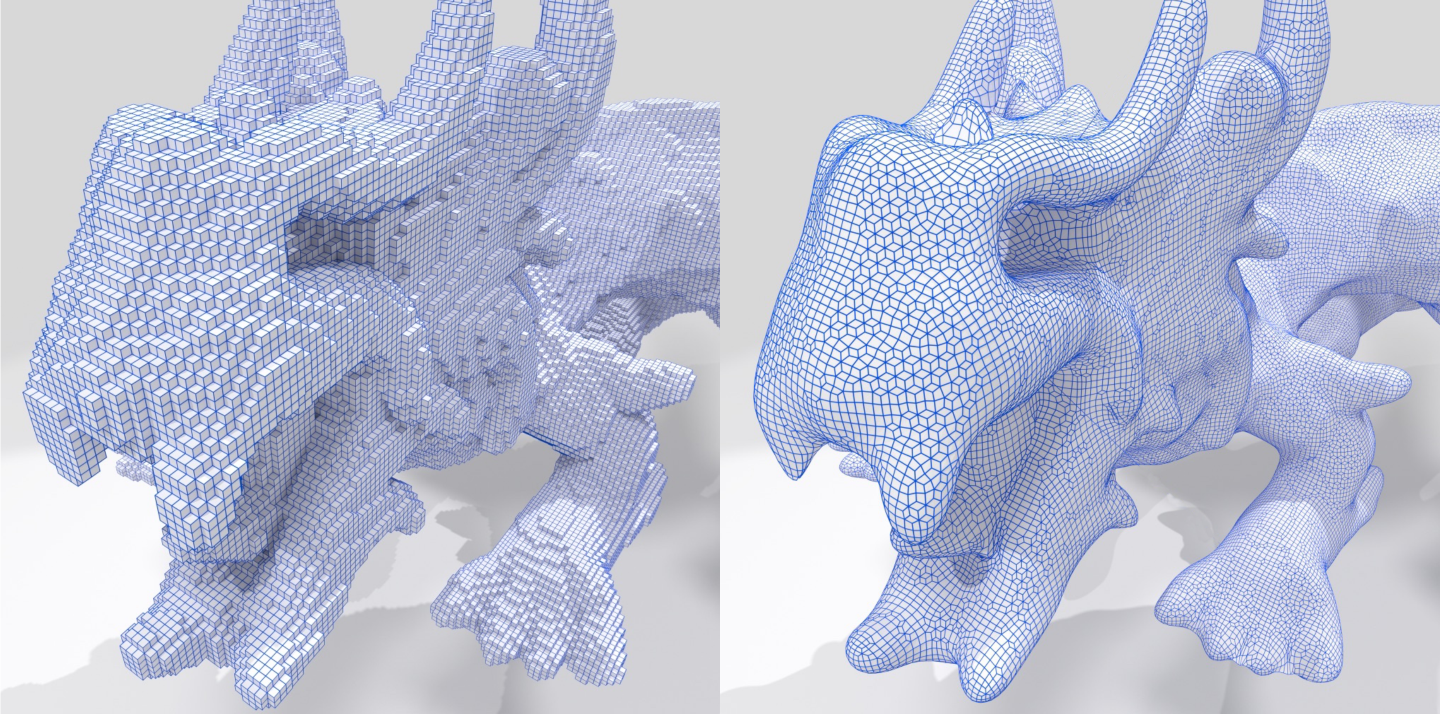
$$\frac{\partial \mathscr{E}(\hat{P})}{\partial \hat{\mathbf{p}}_i} := \alpha \sum_{i=1}^{n} 2(\hat{\mathbf{p}}_i - \mathbf{p}_i) + \beta \sum_{f \in F} \sum_{\hat{\mathbf{e}}_j \in \partial F} 2(\hat{\mathbf{e}}_j \cdot \mathbf{n}_f)\mathbf{n}_f + \gamma \sum_{i=1}^{n} 2(\hat{\mathbf{b}}_i - \hat{\mathbf{p}}_i)$$

⇒ Gradient as a sparse (positive-definite) matrix (linear operator in the vertices position)

Efficient linear solvers to obtain optimal positions P^*: $\nabla \mathscr{E}(P^*) = 0 \Leftrightarrow Ax = b$

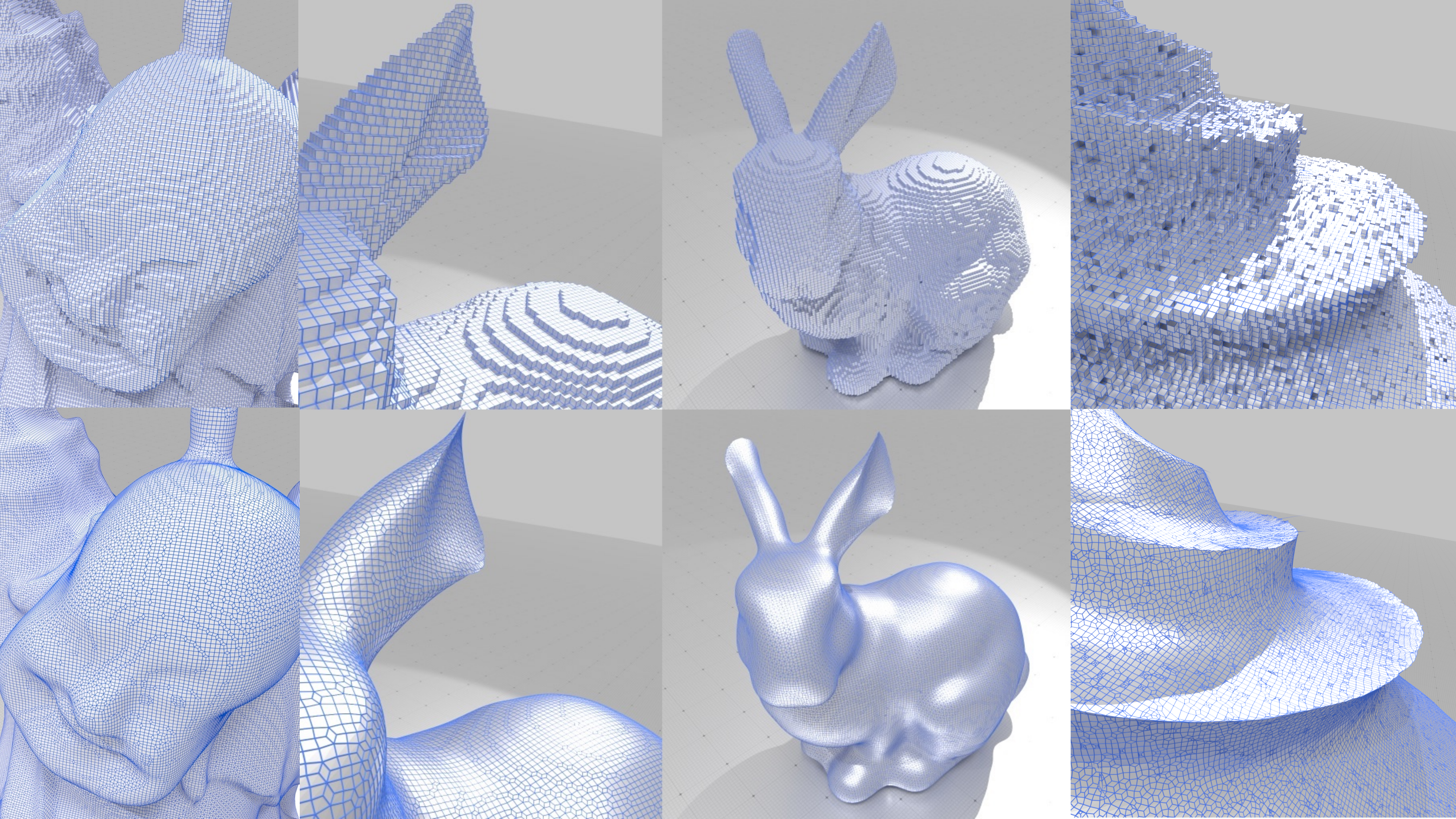
TIMINGS

Dragon 256^3, 104 916 quads.

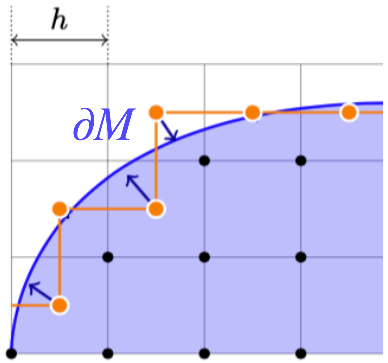


Linear operator construction (4.7s, CPU) and iterative gradient descent on GPU (OpenCL / OpenGL)

3ms per step, ~20 steps for acceptable visual quality,
1.5s for full convergence



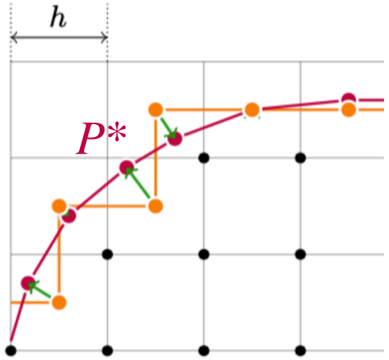
STABILITY RESULTS



If M is smooth (positive reach), $M_h := M \cap (h \cdot \mathbb{Z}^3)$ and for all normal vector fields $\{\mathbf{n}_f\}$

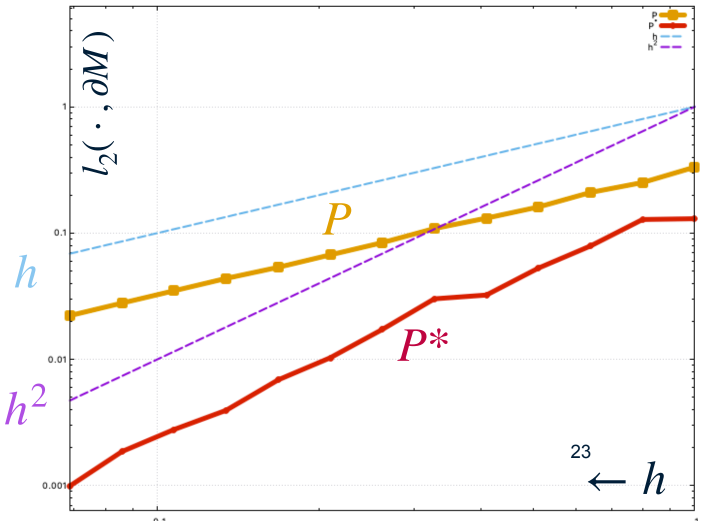
$$\frac{1}{n}\sum_{i=1}^{n} \|\mathbf{p}_i^* - \mathbf{p}_i\| \leq C \cdot h$$

$$\frac{1}{n}\sum_{i=1}^{n} d(\mathbf{p}_i^*, \partial M) \leq C' \cdot h$$



If $\{\mathbf{n}_f\}$ are estimated using a multigrid convergent or a piecewise smooth estimator:

- ⇒ P^* is a better approximation of ∂M than P
- ⇒ regularized quad *normal vectors* are multigrid convergent

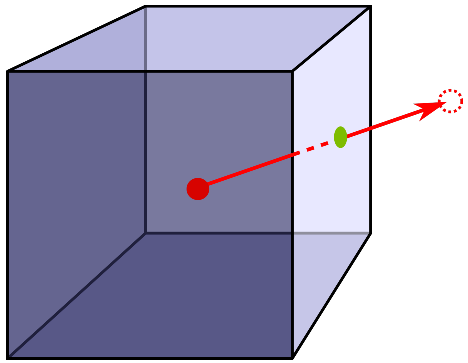


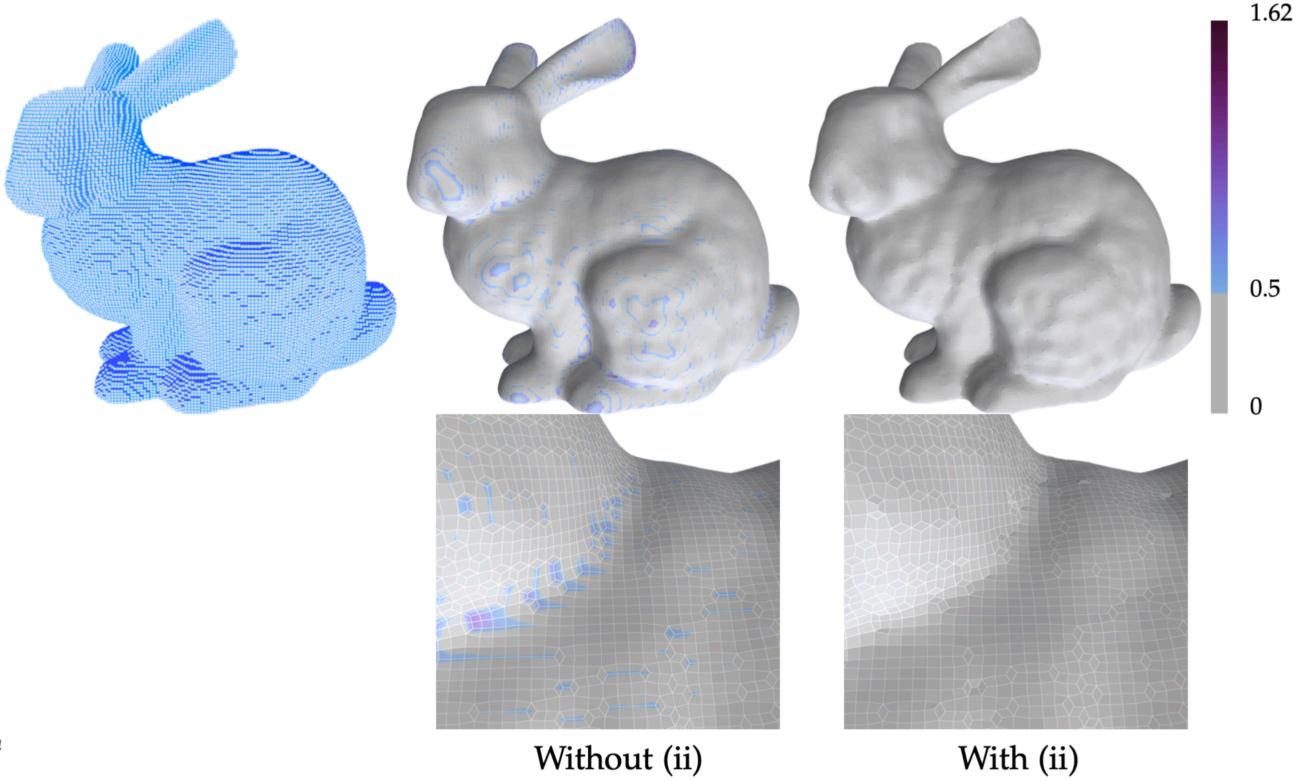
TOPOLOGICAL CONTROL

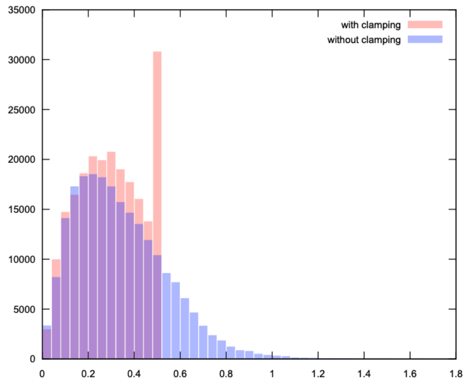
If all points of a face lies in the convex hull of the face vertices, and if each vertex $\mathbf{p}^*$ stays in its $(h - \epsilon)$-cube, the P^* is self-intersection free.

Subspace minimisation as in [HP07]

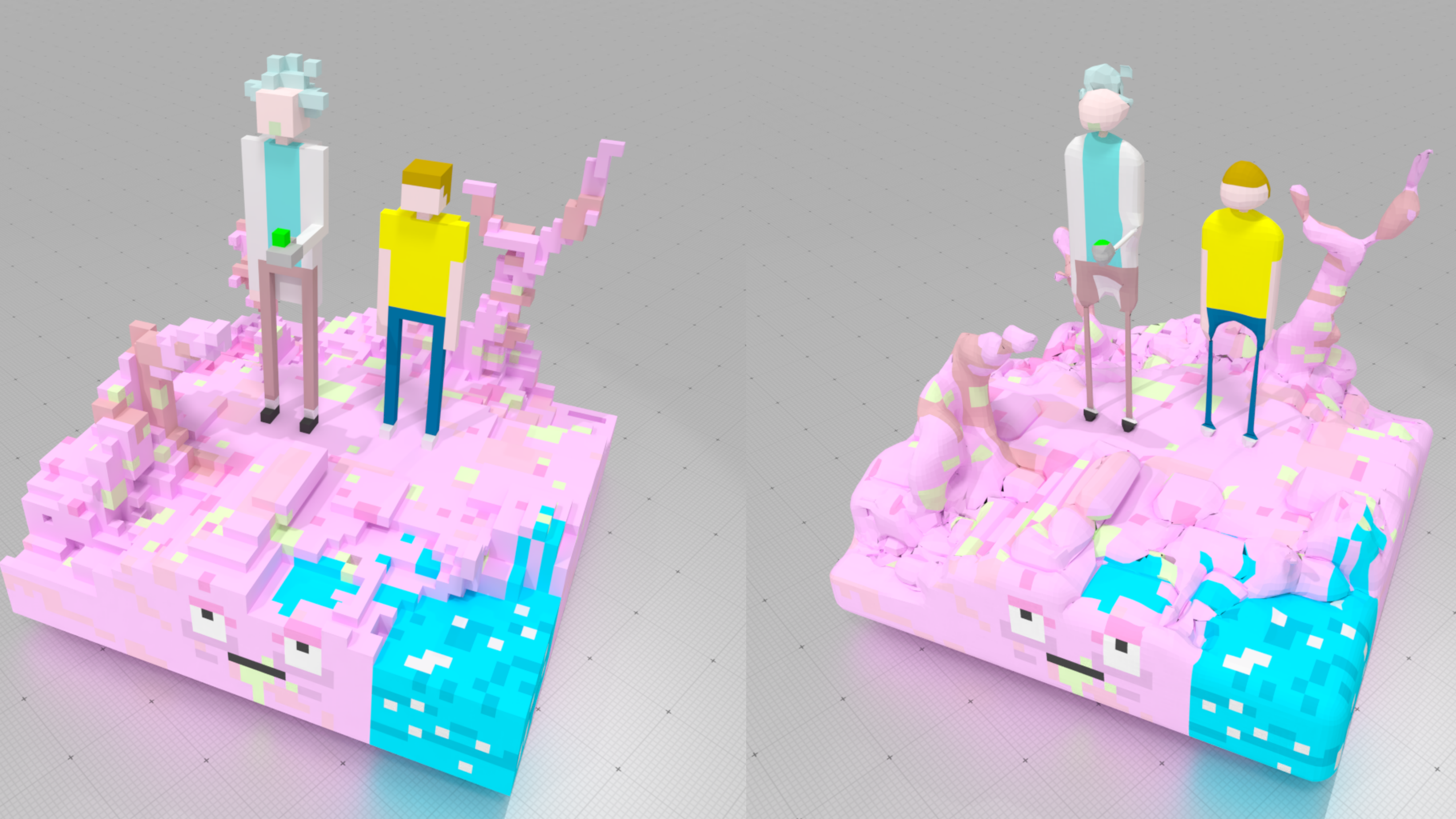
or subgradient scheme with clamping



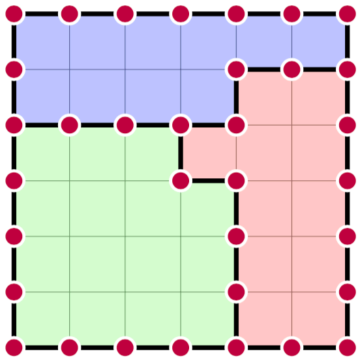


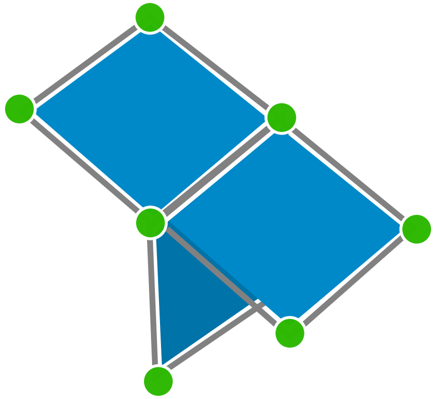


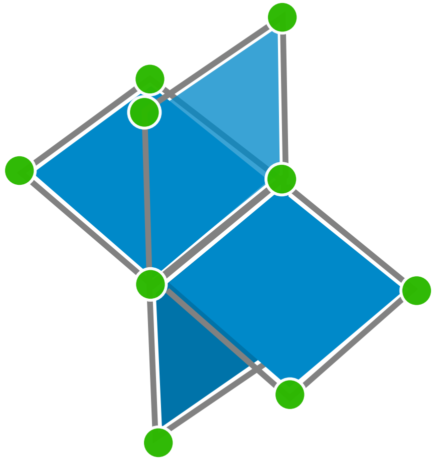
EXTENSIONS



MULTI-LABELED IMAGES







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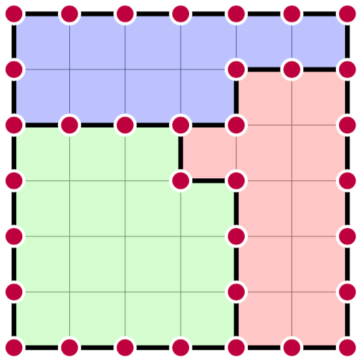
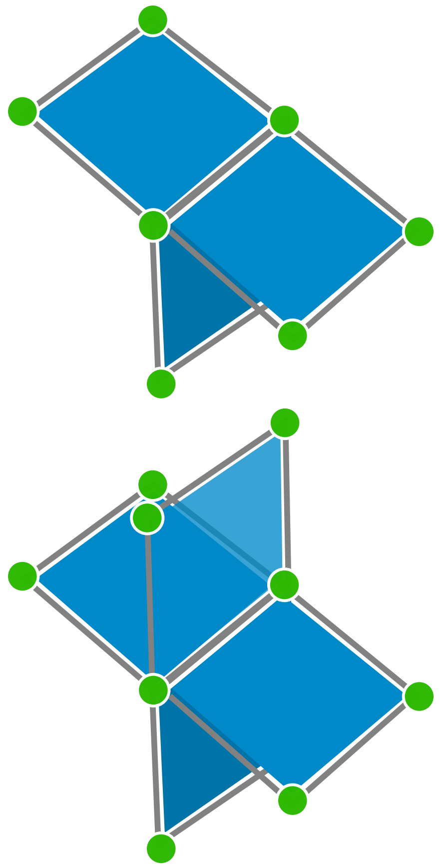
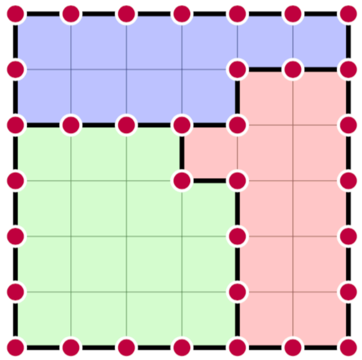


Energy function and gradient operator stay the same !



Data attachment term 🙂

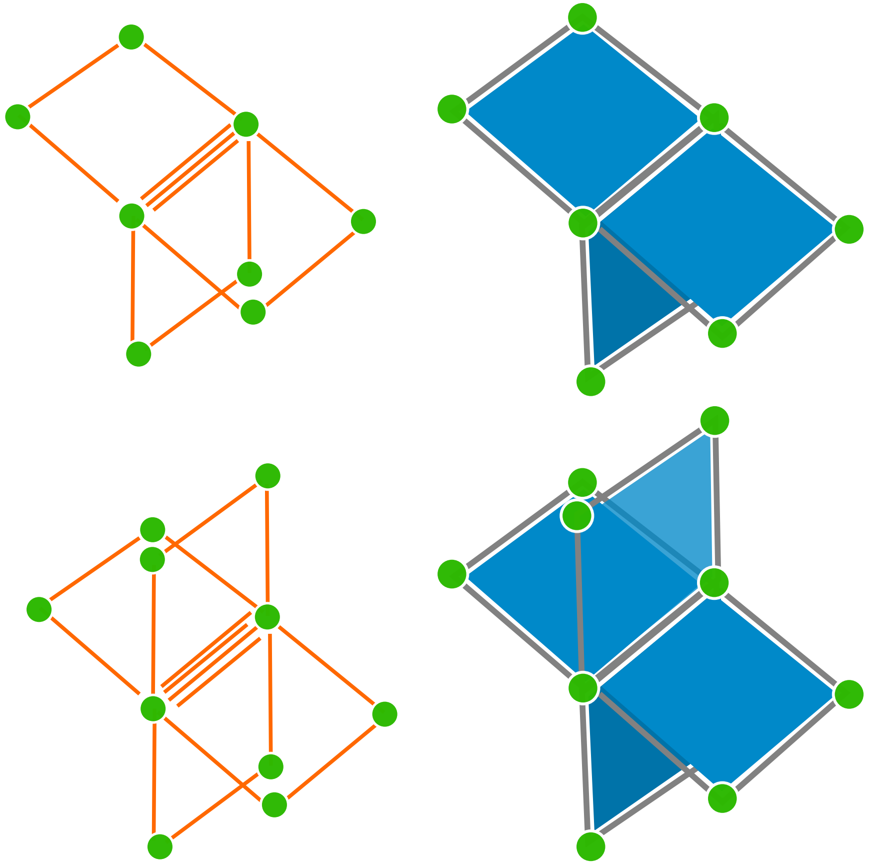
MULTI-LABELED IMAGES



Energy function and gradient operator stay the same !

Data attachment term 🙂

Alignment term 🙂 thanks to the *quad-to-edge* principle



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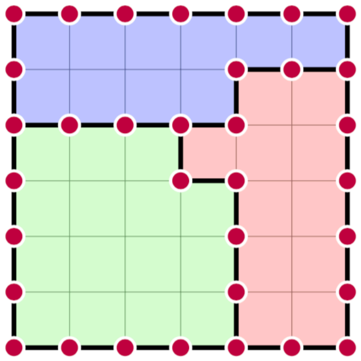

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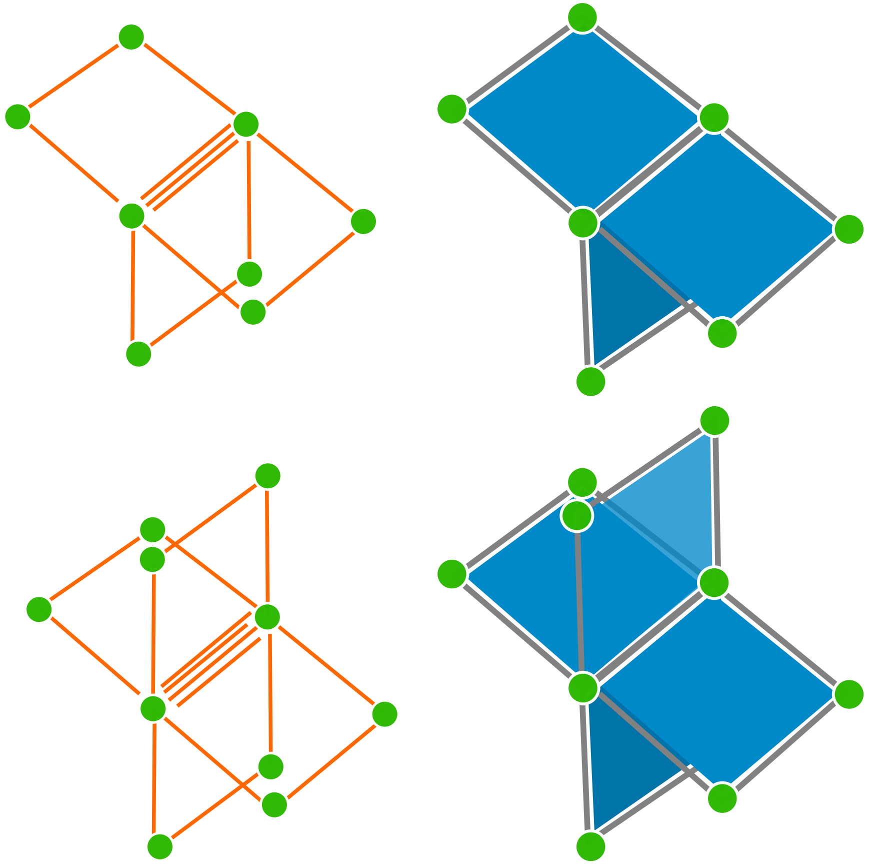
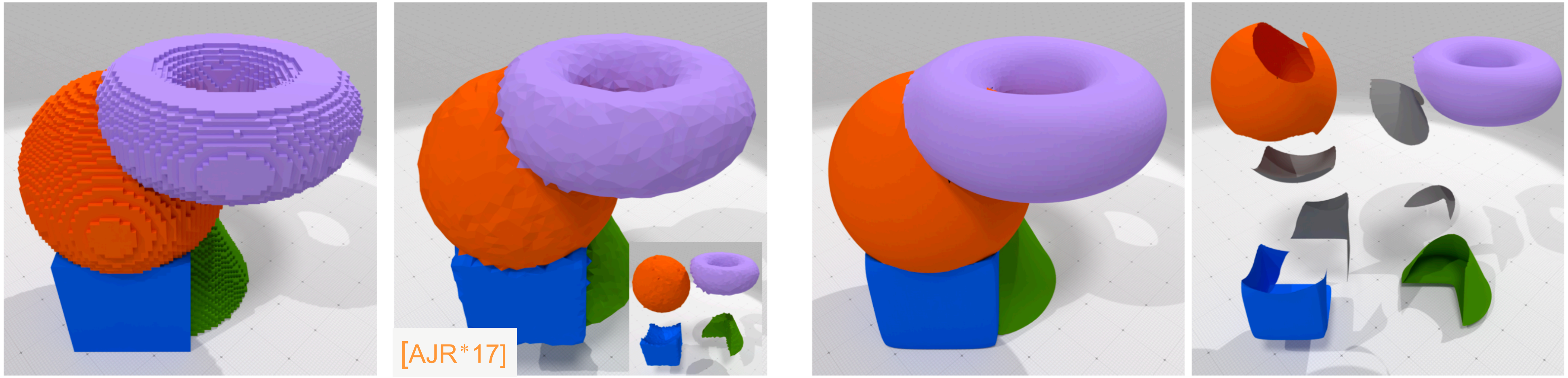


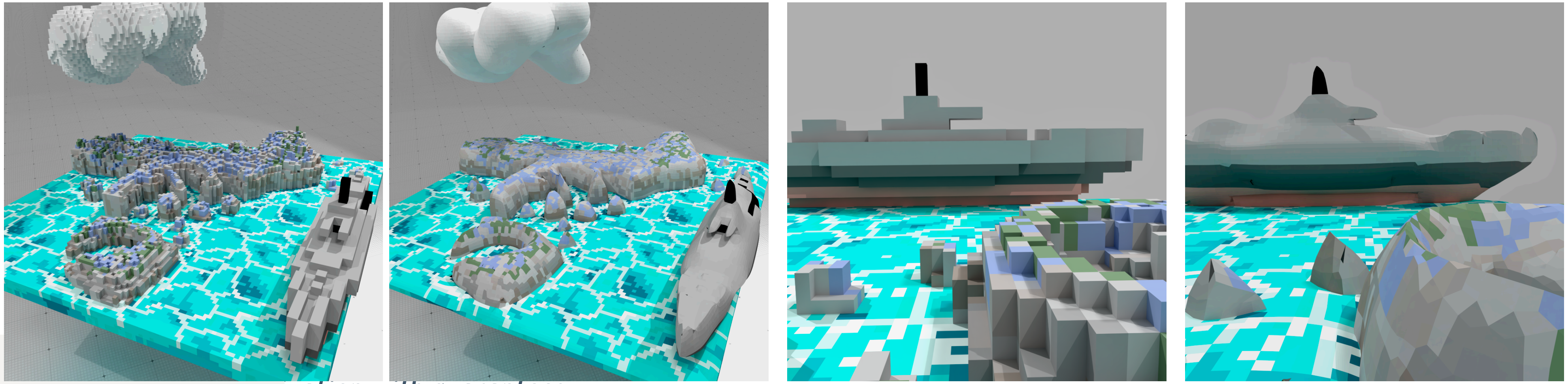
Data attachment term 🙂

Alignment term 🙂 thanks to the *quad-to-edge* principle

Fairness term 🙂 it allows 1-D junction regularization

MULTI-LABELED IMAGES

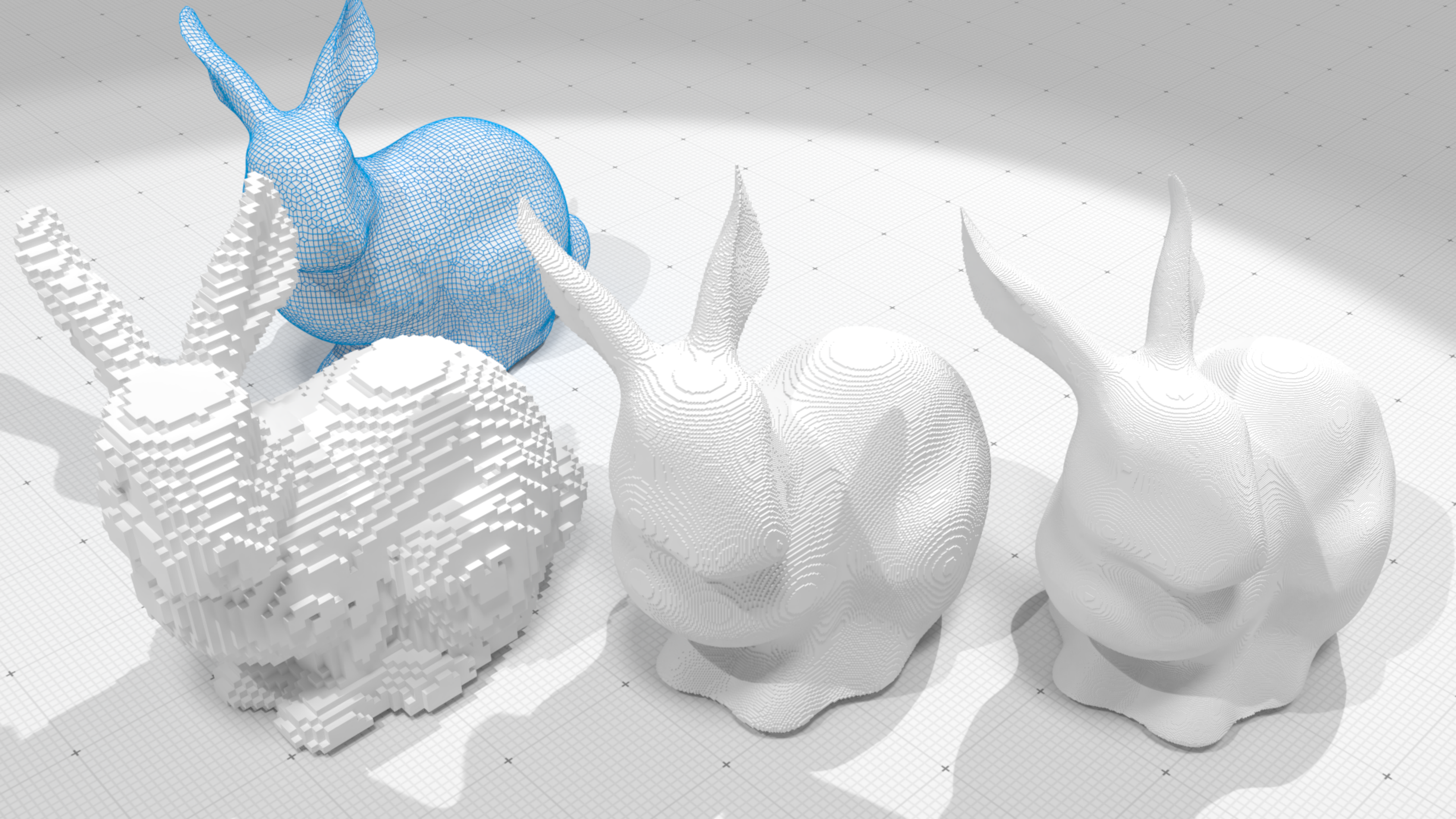




VOXEL UPSCALING

Compute the regularized surface at low voxel resolution

Voxelize the regularized surface at higher resolutions



DISCRETE CALCULUS ON DIGITAL SURFACES

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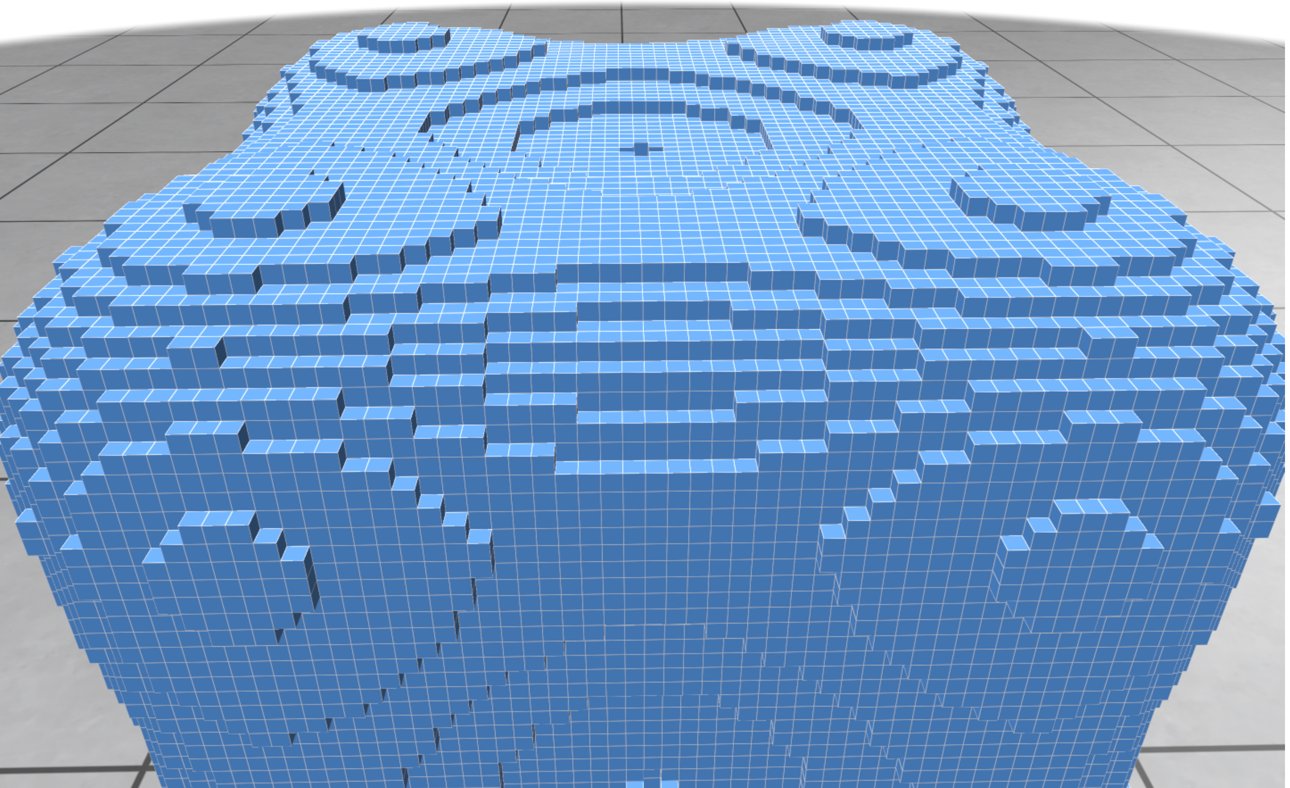

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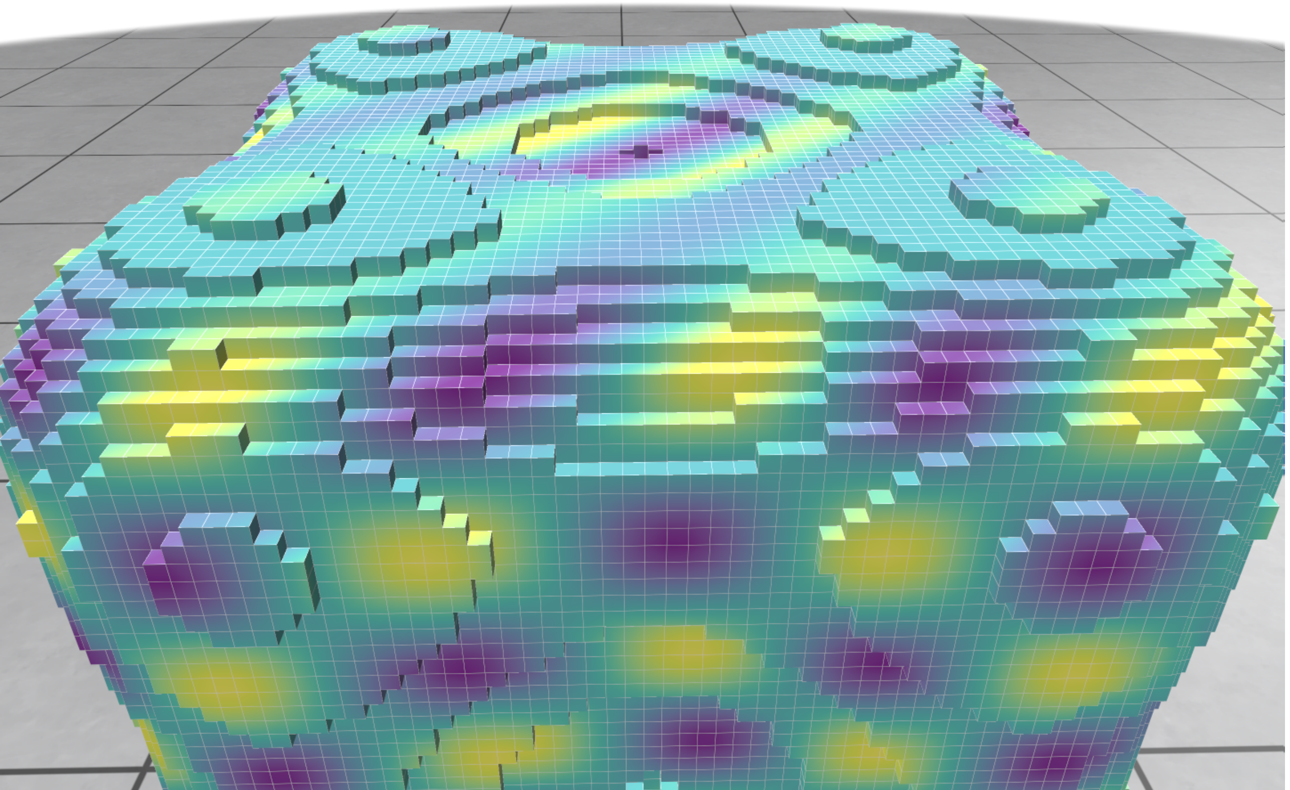
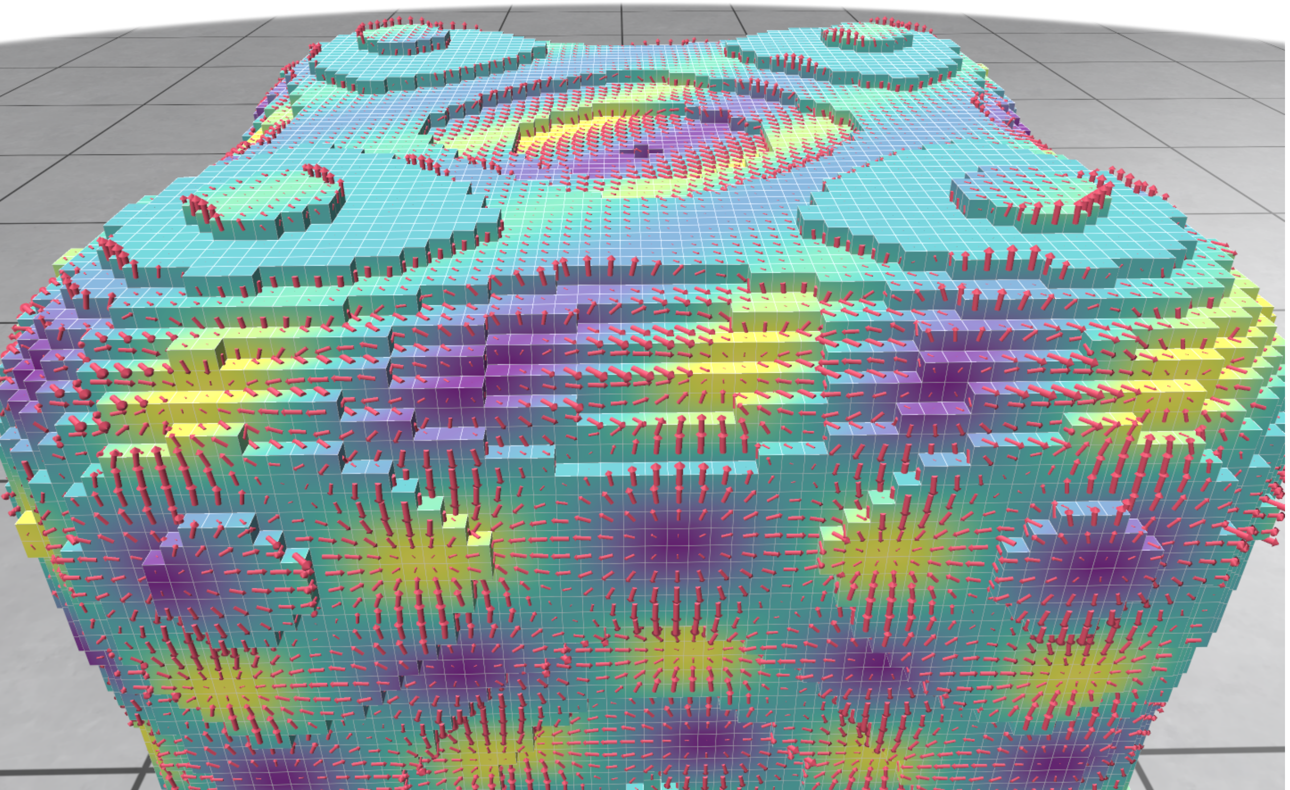


DISCRETE CALCULUS ON DIGITAL SURFACES

[dGBD20]

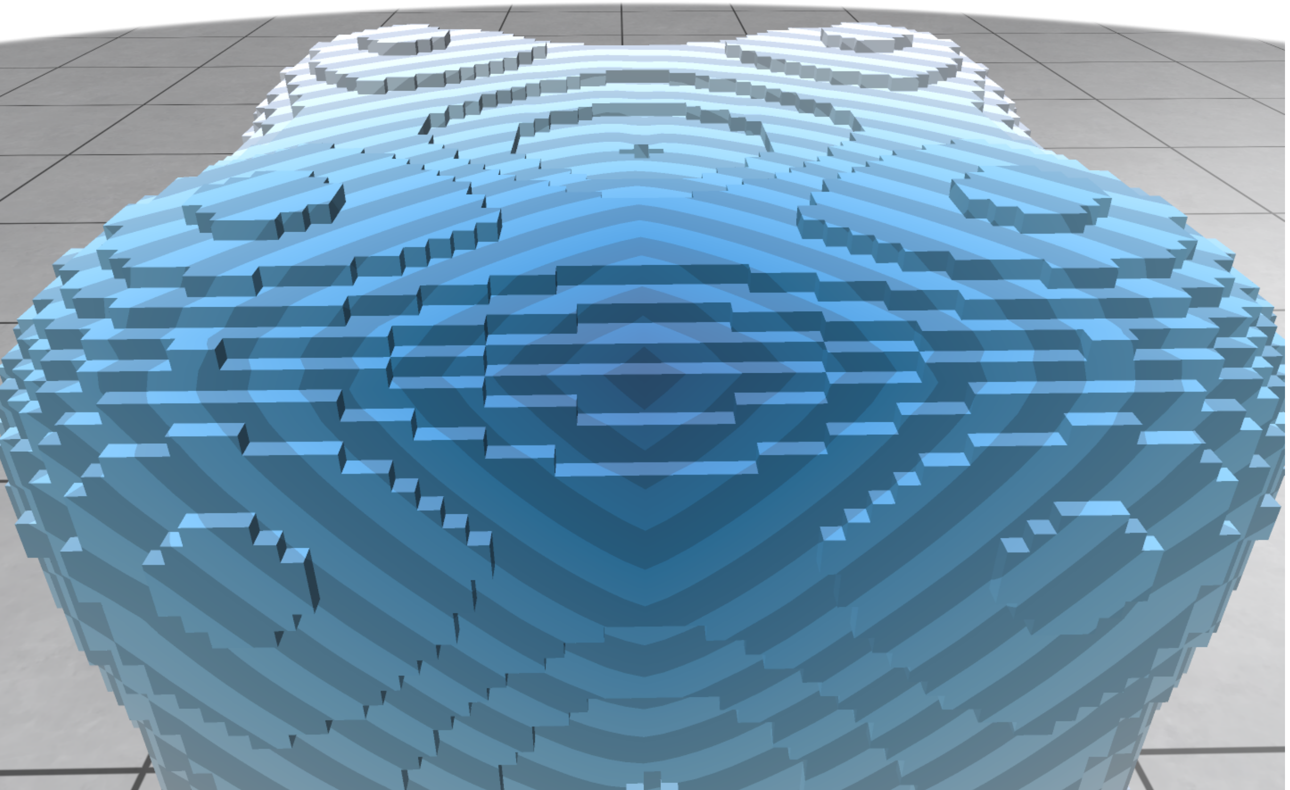
$\nabla, \nabla \cdot, \nabla \times, \sharp, \flat, \Delta \ldots$



DISCRETE CALCULUS ON DIGITAL SURFACES

[dGBD20] + [CWW17]

$\nabla, \nabla\cdot, \nabla\times, \sharp, \flat, \Delta \ldots$



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[dGBD20] + [CWW17]

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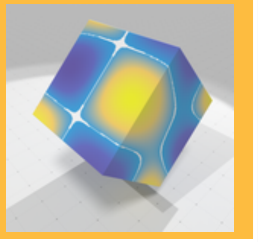
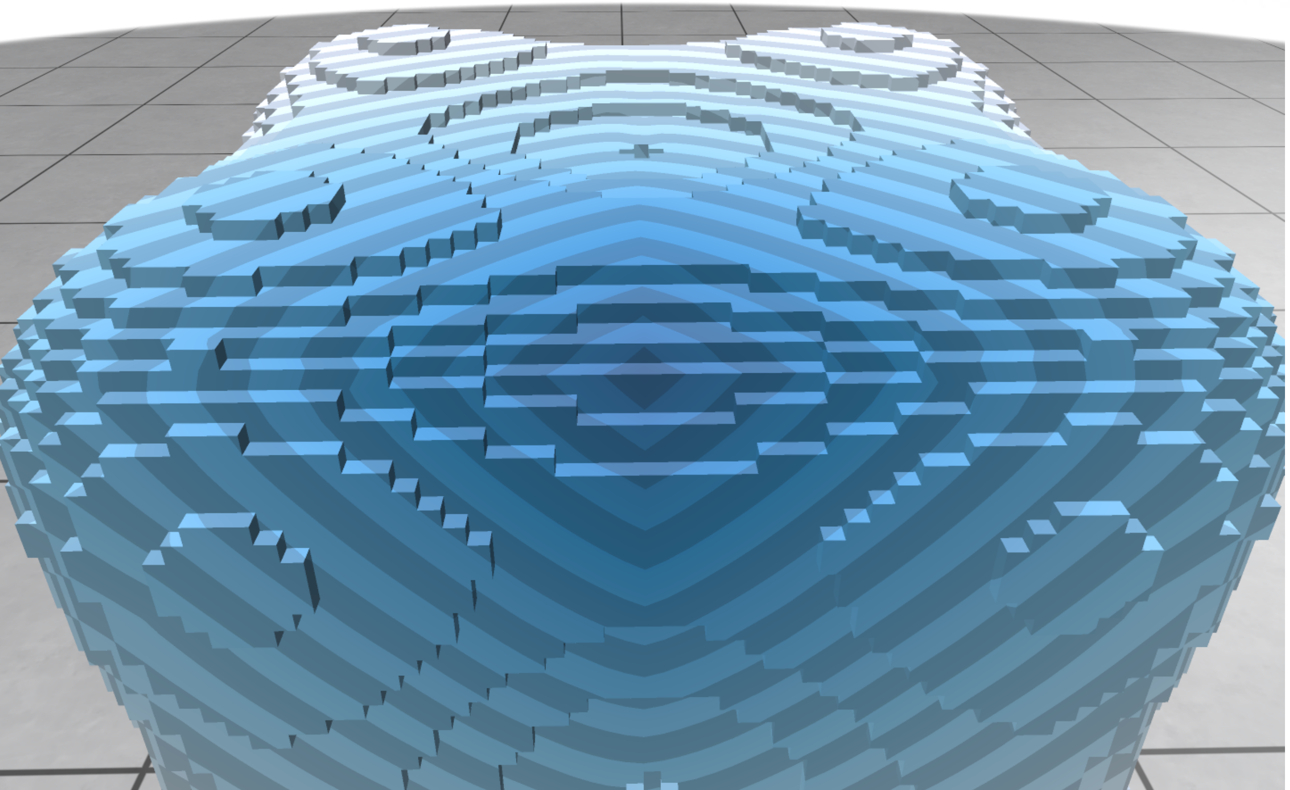
[CCLR19]
Strong Consistency Δ_h





DISCRETE CALCULUS ON DIGITAL SURFACES

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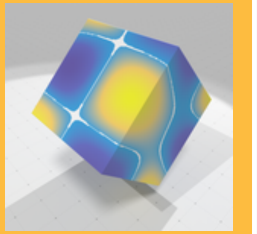

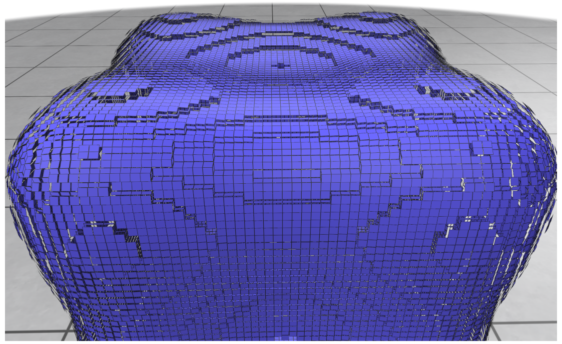

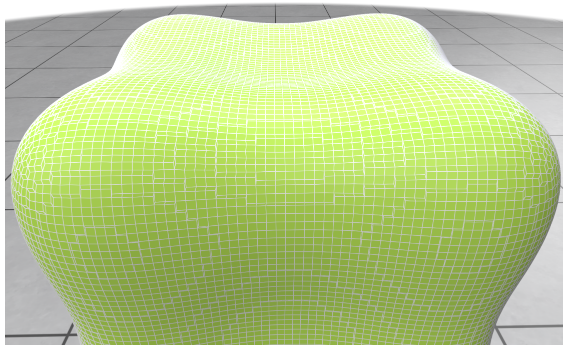

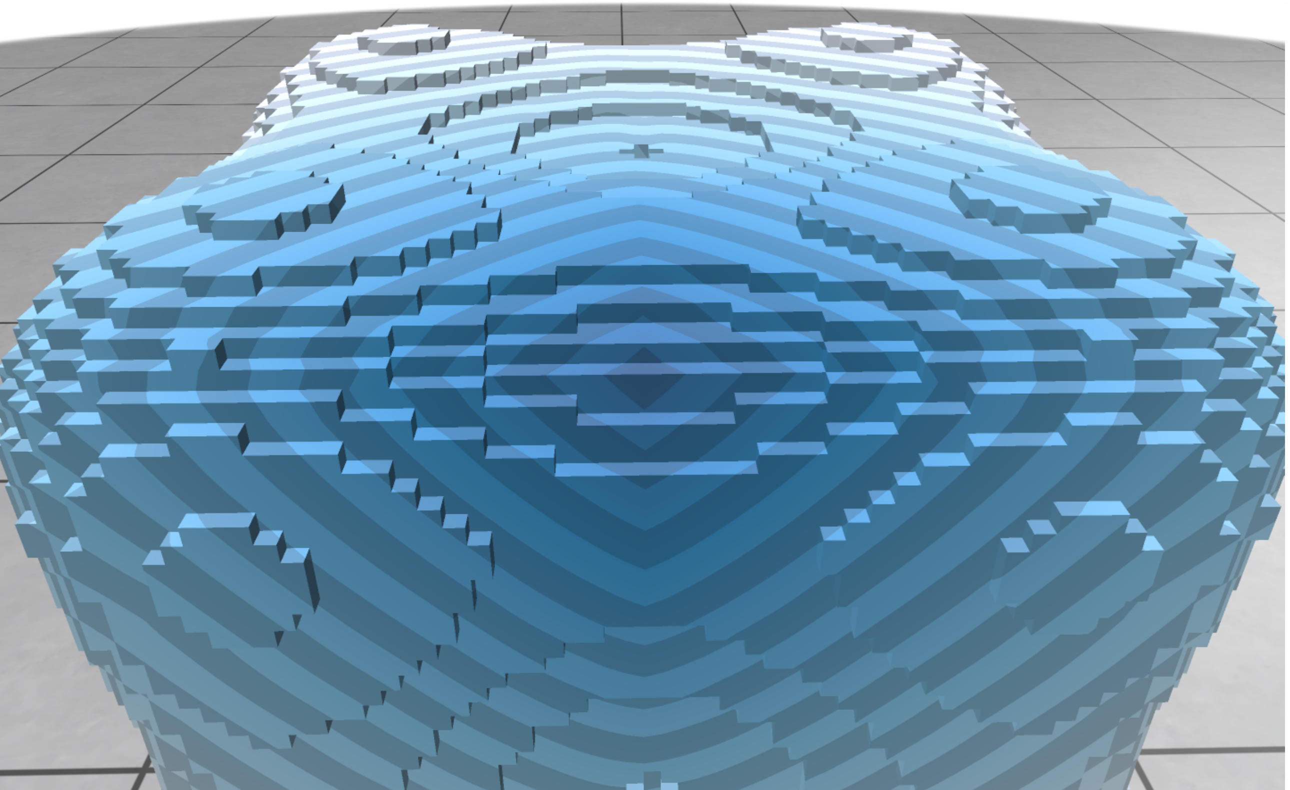

DISCRETE CALCULUS ON DIGITAL SURFACES

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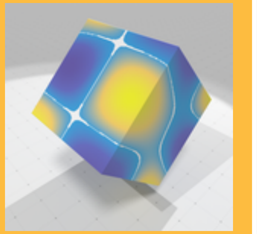

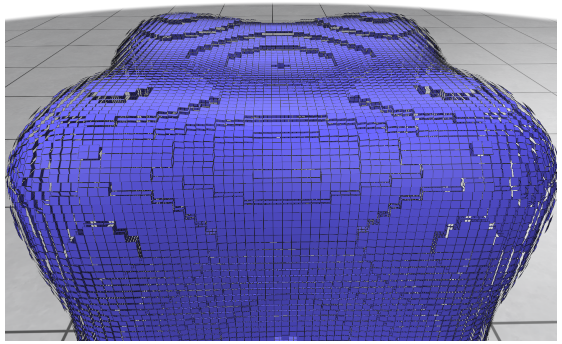

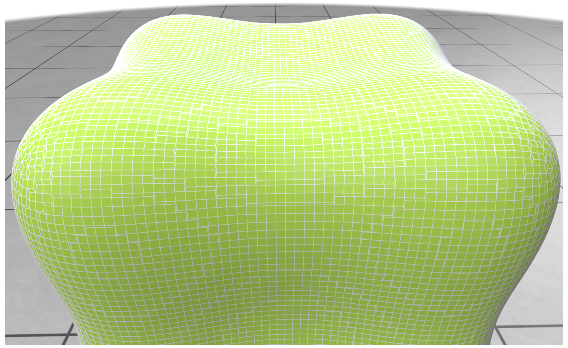

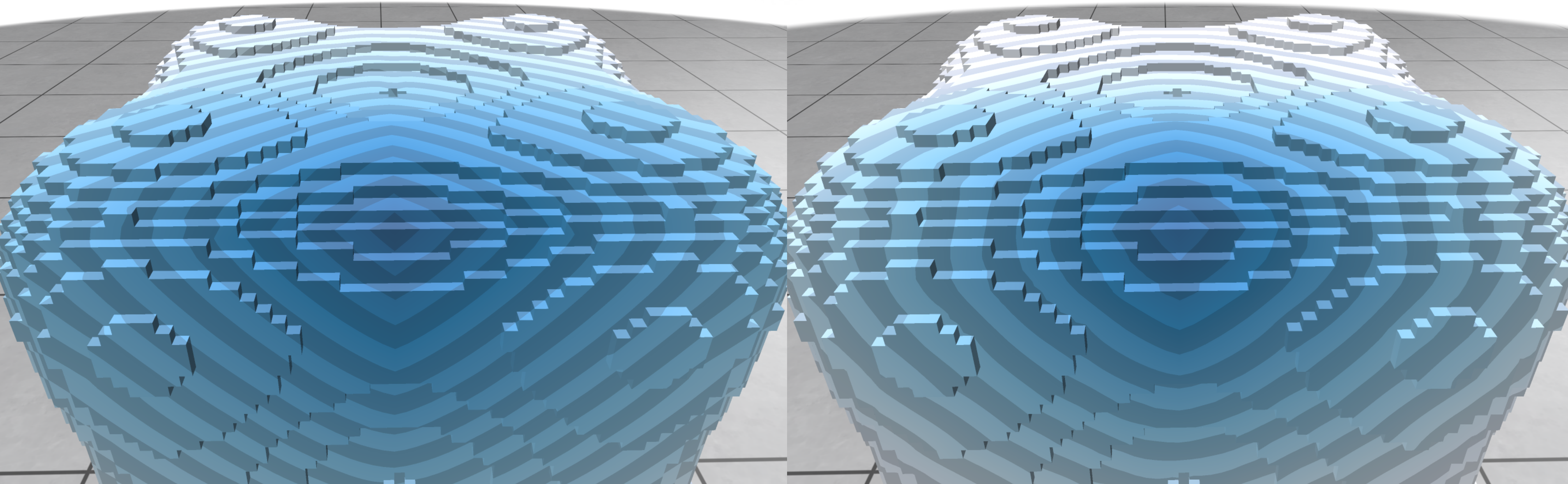

CONCLUSION & FUTURE WORKS

Voxel art regularization tool:

- robust from low to high-res, w/o noise
- easy to implement (convex energy function, GPU solvers)
- one-to-one mapping with input quads
- multi-labeled images
- stability results thanks to digital geometry processing tools

Future works:

- Corrected (embedding, tangent bundle) discrete calculus on digital surfaces

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http://dgtal.org

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