Variational Problems with Conformality Constraints

Ulrich Pinkall*[1]

[1]Technische Universität Berlin – Germany

Abstract

A metric on a discrete surface M assigns a length to each edge. If we are given conformal factors at the vertices and multiply the length of each edge by the two factors at its end points, the resulting new metric is said to be **conformally equivalent** to the old one. If we have a variational energy E for discrete surfaces in three-space (like area, enclosed volume or Willmore functional), we can look for critical points of E under all variations of the surface that change the induced metric only conformally. It turns out that the **Lagrange multiplier** that corresponds to the conformality constraint can be interpreted as the discrete ananlog of a holomorphic quadratic differential on M.
Minimizers of geometric energies under a conformality constraint can be practically useful for a variety of modelling tasks. They are a two-dimensional analog for elastic curves (minimizers of bending energy under a length constraint). Such discrete surfaces can be practically useful for a variety of modeling tasks.

*Speaker