Discrete Vector Bundles with Connection and the Bianchi Identity

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Abstract

Discrete de Rham complexes have proven to be beneficial in the discovery and analysis of new finite element spaces as well as helping to understand the relationship between discrete exterior calculus and finite element exterior calculus. In order to build similar discrete frameworks for differential geometry one needs complexes where the failure of d^2 being zero is related to the curvature of the connection. We develop a discrete / combinatorial formulation of the metric free part of discrete vector bundles with connection which is natural with respect to appropriate mappings of the base space. The main objects are discrete cochains obtained with a new type of de Rham map from smooth vector bundle valued forms. The central operators are a discrete covariant exterior derivative and a combinatorial wedge product. We demonstrate the key properties of these operators and show that they are natural with respect to the mappings referred to above. We characterize when a discrete vector bundle with connection is trivializable or has a trivial lower rank subbundle. This machinery is used to define a discrete curvature as certain linear maps and we show that our formulation satisfies a discrete Bianchi identity.

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