A generalized conditional gradient method for dynamic inverse problems with optimal transport regularization

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Abstract

In this talk we present a new dynamic generalized conditional gradient approach for solving dynamic inverse problems with optimal transport regularization.

We consider a dynamic inverse problem comprised of a fidelity term, penalizing the pointwise in time discrepancy between the observation and the unknown, and a regularizer keeping track of the dynamics, given by the Benamou-Brenier energy constrained via the homogeneous continuity equation. Such model is suitable, for example, for reconstructing the trajectory of moving particles subjected to undersampled and noisy measurements.

Employing the characterization of the extremal points of the Benamou-Brenier energy we define the atoms of the problem as measures concentrated on absolutely continuous curves in the domain. We then introduce our dynamic generalization of a conditional gradient method that consists in iteratively adding suitably chosen atoms to the current sparse iterate and subsequently optimize the coefficients in the resulting linear combination. We show that the method converges with a sublinear rate to a minimizer of the objective functional. Additionally, we propose heuristic strategies and acceleration steps that allow to implement the algorithm efficiently. Finally, we present numerical examples that demonstrate the effectiveness of our algorithm and model at reconstructing heavily undersampled dynamic data, together with the presence of noise.

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