Discrete Exterior Calculus framework and proximal alternating minimization for solving discrete Mumford-Shah and Amborsio-Tortorelli models

Hoangtrieuvy Le*[1], Marion Foare*[2], Mamadou Ngom*[3], and Nelly Pustelnik*[4]

[1]Univ Lyon, Ens de Lyon, Univ Lyon 1, CNRS, Laboratoire de Physique – Univ Lyon, Ens de Lyon, Univ Lyon 1, CNRS, Laboratoire de Physique – France
[2]Univ Lyon, Ens de Lyon, Univ Lyon 1, CNRS, INRIA, LIP, Lyon – CNRS, INRIA, LIP, Lyon – France
[3]Univ Lyon, Ens de Lyon, Univ Lyon 1, CNRS, Laboratoire de Physique – ENS de Lyon, Univ Lyon, Ens de Lyon, Univ Lyon 1, CNRS, Laboratoire de Physique – France
[4]Univ Lyon, Ens de Lyon, Univ Lyon 1, CNRS, Laboratoire de Physique – Univ Lyon, Ens de Lyon, Univ Lyon 1, CNRS, Laboratoire de Physique – France

Abstract

This work is dedicated to joint image restoration and contour detection. We consider both the discrete Mumford-Shah and the Ambrosio-Tortorelli models, which can be formulated as a minimization involving the sum of a data fidelity term, a term enforcing smoothness everywhere in the image except at the location of the contours, and a third term that penalizes the length of the contours, being the discrete counterpart of the 1D Hausdorff measure. A particular attention is paid for the derivation of the coupling and the penalization terms relying on the derivative operator and expressed by means of the Discrete Exterior Calculus framework. In this work, we derive proximal alternating minimization schemes with convergence guarantees (PALM-AT and SL-PAM) to estimate a critical point of our energy. We compare their behaviors when the discrete counterpart of the 1D Hausdorff measure is modeled either by an l1-norm or Ambrosio-Tortorelli penalization. The closed-form expressions of the involved proximity operators are provided. A thorough numerical study is conducted to evaluate the performance of both numerical schemes as well as comparisons between l1-norm and Ambrosio-Tortorelli penalization.

*Speaker